Mathematical Tables and other Aids to Computation

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and Other Aids to Computation
by

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A Symposium of Large Scale Digital Calculating Machinery

Under the joint sponsorship of Harvard University and the Bureau of Ordnance, United States Navy, 336 representatives of university, industrial and government laboratories and research groups met in Cambridge, Mass. from January 7–10, 1947 to participate in a four-day Symposium on Large Scale Digital Calculating Machinery.

The meetings were held in Harvard's new Computation Laboratory and, in part, the Symposium celebrated the formal opening of this new facility.

In addition to the extensive group of technical papers, the program included a demonstration of the Automatic Sequence Controlled Calculator (usually referred to as the Mark I Calculator), and a preview of the Mark II Calculator which is now being assembled at Harvard for the Bureau of Ordnance. The latter machine will later be reassembled at the Dahlgren Proving Ground of the Navy.

The program of the Symposium was as follows:

I. Tuesday, 7 January, 10:00 a.m.

Opening Addresses by

Edward Reynolds, Administrative Vice-President of Harvard University

Rear Admiral C. T. Joy, USN, Naval Proving Ground, Dahlgren, Virginia

Prof. H. H. Aiken, Technical Director of the Computation Laboratory, Harvard University

Inspection of the Computation Laboratory and of Mark II Calculator

II. Tuesday, 7 January, 2:00 p.m.

Existing Calculating Machines, Prof. W. E. Bleick, Chairman

1. "The Work of Charles Babbage" by Mr. R. H. Babbage[1]
2. "Mark I Calculator" by Mr. R. M. Bloch
3. "Brief description and operating characteristics of the ENIAC" by Dr. L. P. Tabor
4. "Bell Telephone Laboratories relay computing systems" by Mr. S. B. Williams
5. "Mark II Calculator" by Mr. R. V. D. Campbell

III. Wednesday, 8 January, 9:30 a.m.

The Logic of Large Scale Calculating Machines, Prof. W. H. Furry, Chairman

1. "Problems of mathematical analysis involved in machine computations" by Dr. A. W. Wundheiler
2. "The organization of large scale calculating machinery" by Dr. G. R. Stibitz

IV. Wednesday, 8 January, 2:00 p.m.

Storage Devices, Dr. J. H. Curtiss, Chairman

1. "Mercury delay lines as a memory unit" by Dr. T. K. Sharpless
2. "Slow electromagnetic waves" by Prof. L. N. Brillouin
3. "High Speed electrostatic storage" by Mr. J. W. Forrester
4. "Magnetic and phosphor coated disks" by Dr. B. L. Moore
5. "The selectron—a tube for selective electrostatic storage" by Dr. Jan Rajchman
6. "Optical and photographic storage techniques" by Dr. A. W. Tyler

V. Thursday, 9 January, 9:30 a.m.

Numerical Methods and Suggested Problems for Solution, Dr. Mina S. Rees, Chairman

1. "Method of finite differences for the solution of partial differential equations" by Prof. Richard Courant

2. "On computational techniques for certain problems in fluid dynamics" by Dr. R. J. Seeger
3. "Computational problems arising in connection with economic analysis of interindustrial relationships" by Prof. W. W. Leontief
4. "On the accumulation of errors in processes of integration on high speed calculating machines" by Prof. H. A. Rademacher
5. "Fluid mechanics computations" by Prof. H. W. Emmons
6. "Firing tables" by Dr. L. S. Dederick

VI. Thursday, 9 January 2:00 p.m.

Sequencing, Coding, and Problem Preparation, Prof. J. A. Stratton, Chairman

1. "Coding for large scale calculating machinery" by Dr. H. H. Goldstine
2. "Preparation of problems for EDVAC-type machines" by Dr. J. W. Mauchly
3. "The preparation of problems for large scale calculating machinery" by Mr. J. O. Harrison, Jr.

VII. Friday, 10 January, 9:30 a.m.

Input and Output Devices, Prof. E. L. Chaffee, Chairman

1. "Application of printing telegraph techniques to large scale calculating machinery" by Mr. F. G. Miller
2. "Some physical aspects of magnetic recording" by Mr. Otto Kornei
3. "The numeroscope" by Mr. H. W. Fuller
4. "Input and Output Devices for electronic digital calculating machinery" by Mr. S. N. Alexander
5. "An input device using multiple gates" by Dr. Morris Rubinoff
6. "Photographic methods of handling input and output data" by Dr. K. G. Macleish
7. "Transfer between external and internal memory" by Mr. C. B. Sheppard

VIII. Friday, 10 January, 2:00 p.m.

Conclusions and Open Discussion, Prof. C. C. Bramble, Chairman

1. "Publication, classification, and patents" by Prof. S. H. Caldwell
2. "Le domaine du calcul mécanique" by Dr. Louis Couffignal (paper not read, but to be included in the published report).
3. "New Vistas in mathematics" by Dr. A. T. Waterman

All of the remarks and papers of the 34 speakers, as well as the related discussions, are to be published in a later volume of the *Annals* of the Computation Laboratory.

Both Harvard University and the Bureau of Ordnance are to be congratulated for their enterprise in making possible this significant meeting. The extensive program reflected every phase of the sweeping progress being made in the field of large-scale calculators, and the large attendance offered a unique opportunity for the numerous small group meetings which do much to facilitate the exchange of helpful information.

The following members registered for the Symposium:

Milton Abramowitz, mathem., NBSMTP
Erik Ackerlind, group leader, Northrop Aircraft Corp., Hawthorne Field, Cal.
L. V. Ahlfors, prof. math., Harvard Univ.
H. H. Aiken, prof. appl. math., Harvard Univ.
L. W. Alberts, grad. stu., Harvard Law Sch.
S. N. Alexander, group leader, computer components group, National Bureau of Standards (NBS)
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E. G. Andrews, engin., Bell Tel. Lab., New York
R. C. Archibald, prof. math., Brown Univ.

George Arfken, Jr., grad. stu., Yale Univ.
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Dr. A. M. Turing, research fellow, National Res. Lab., Teddington, England
Dr. A. W. Tyler, physicist, Eastman Kodak Co., Rochester, N. Y.
C. F. Van Aken, chief machine tabul. div., Bureau of Census, Washington
Capt. A. M. Van Eaton, commanding officer, Office Naval Res., Boston
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Henry Wallman, assoc. prof. math., Mass. Inst. Techn.
John Walsh, electr. engin., Watson Lab., AAF AMC, Cambridge
J. H. Warden, president, The Autocall Co., Shelby, O.
K. L. Warthman, engin., Bell Tel. Lab., New York
Dr. A. T. Waterman, chief sci. planning div., Navy Dept., Office Naval Res., Wash.
J. R. Weiner, engin., Raytheon Mfg. Co., Waltham, Mass.
Albert Wertheimer, head engin., Navy Dept., Bur. Ord., Washington
Dr. F. J. Weyl, mathem., Navy Dept., Bur. Ord., Wash.
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Dr. A. W. Wundheiler, mathem., Navy Dept., Bur. Ord., Wash.
Jeff Wylie, bureau head, Boston Office, *Time Mag.*
Patrick Youtz, res. assoc. electr. engin., Mass. Inst. Techn.
Dr. V. K. Zworykin, director electronic res., Radio Corp. Amer., Princeton, N. J.

S. H. C.

[1] Richard Henry Babbage (1883–   ) is a son of Henry Whitmore B. (1855–1911), son of Henry Prevost B. (1824–1918), son of Charles Babbage (1792–1871). He is an assistant editor of Canada's national farm magazine, *Family Herald and Weekly Star.*

Appendix

A. Existing and Practically completed Digital Computing Machines developed by or presently on loan or contract to the Government.

I. *Department:* War
Branch or Bureau: Ordnance Dept.
Permanent Location of Machine: Ballistic Research Lab., Aberdeen Proving Ground, Md.
Type: A. Relay (small) manufactured by IBM Corp.
B. Relay (small) manufactured by IBM Corp.
C. Electronic (ENIAC) manufactured by Moore School of Electrical Engineering, University of Pennsylvania. See *MTAC*, v. 2, p. 97f.
D. Relay (large) manufactured by Bell Telephone Labs.

II. *Department:* War
Branch or Bureau: Army Ground Forces
Permanent Location of Machine: Army Ground Forces, no. 4, Fort Bliss, Texas.
Type: Relay (small) manufactured by Bell Telephone Labs.

III. *Department:* Navy
Branch or Bureau: Office of Naval Research
Permanent Location of Machine: Naval Research Lab., Bellevue, Md.
Type: Relay (small) manufactured by Bell Telephone Labs.

IV. *Department:* Navy
Branch or Bureau: Bureau of Ordnance
Permanent Location of Machine: Harvard Computation Lab., Cambridge, Mass.
Type: Electromechanical (IBM Sequence Controlled Calculator = "Mark I Calculator"). See *MTAC*, v. 2, p. 185f.

V. *Department:* Navy
Branch or Bureau: Bureau of Ordnance
Permanent Location of Machine: Navy Proving Ground, Dahlgren, Va. [In probable operation by July 1947].
Type: A. Relay (small) manufactured by IBM Corp.
B. Relay (large) "Mark II Calculator," manufactured by the Computation Lab., Harvard Univ. [Editorial Note: It is about 12 times faster than Mark I, and cost about $400 000.]

VI. *Department:* National Advisory Committee for Aeronautics
Permanent Location of Machine: Langley Memorial Lab., Langley Field, Va.
Type: Relay, manufactured by Bell Telephone Labs. (under contract expiring in 1948).

B. Present Automatic Digital Computing Machine Construction and Development Projects supported entirely or in part by the Government.

I. *Department:* War
Branch or Bureau: Ordnance Department
Description of activity:
A. Moore School of Electrical Engineering, Construction of an electronic digital machine ("EDVAC").
B. Institute for Advanced Study and RCA Labs., Princeton, N. J. Construction of an electronic digital machine (financed only in part by federal funds).
C. National Bureau of Standards, Washington, D. C., Long-range component development program.

II. *Department:* Navy
Branch or Bureau: Office of Naval Research
Description of Activity:
A. National Bureau of Standards, Construction of an electronic digital machine.
B. Servomechanisms Lab., Mass. Institute of Technology, Construction of an electronic digital machine to be used in a large guided-missile flight simulator.

III. *Department:* Navy
Branch or Bureau: Bureau of Ordnance
Description of Activity:
A. Harvard Computation Lab., Research and preparation of specifications for an electronic digital machine.
B. Naval Ordnance Lab., White Oaks, Md., Construction of an electronic digital machine, temporarily abandoned (Dec. 1946).

IV. *Department:* Commerce
Branch or Bureau: Bureau of the Census
Description of Activity: National Bureau of Standards, Construction of an electronic digital machine.

J. H. CURTISS

National Bureau of Standards

EDITORIAL NOTE: Other projected digital machines, unaided in their construction by the U. S. Government, are being built by:

(i) The Eastman Kodak Co., Rochester, N. Y.
(ii) The University of California, Berkeley.
(iii) The National Physical Laboratory, Teddington, England, under the direction of Dr. ALAN M. TURING. The planned Automatic Computing Engine will work at the speed of the ENIAC or possibly somewhat higher, and will take advantage of new technical developments, making possible both a greater memory capacity and a higher degree of complexity in the instructions.

P. G. Scheutz, Publicist, Author, Scientific Mechanician, and Edvard Scheutz, Engineer,—Biography and Bibliography

PEHR GEORG SCHEUTZ (1785–1873), son of a tavern-keeper, after passing examinations in law, practised in different localities before settling in Stockholm in 1812. In 1817 as owner of a printing establishment founded by the well-known writer F. CEDERBORGH, Scheutz devoted himself from that time principally to literary interests. He soon became part owner and co-editor of a newspaper which, with changed name *Argus* (1820–1836), became Sweden's most important political newspaper in the 1820's. During 1826–

1842 he published five other journals and newspapers devoted to manufactures and management; art, sloyd, and kindred sciences; engineering; industry and trade. In the field of literature he worked as a translator of Shakespeare, Werner, Walter Scott, Boccaccio, and others; his editions of Merchant of Venice (1820) and of Julius Caesar (second ed., 1831), the first translations of Shakespeare into Swedish, are in the Library of Harvard University. (His unpublished translation of King Lear was performed at Lindeberg's Theatre). He also published numerous handbooks, and books of instruction. In 1842 he gave up his printing establishment and became one of the regular staff of the *Aftonbladet* (Evening News) Stockholm, with special interest in technical and economic subjects, and continued this interest until a few years before his death (Nos. 24, 25).

But it was neither as a literary man nor as a publicist that Scheutz achieved his greatest commendation, but rather as inventor of a calculating machine. The Difference Engine of CHARLES BABBAGE (1792–1871) had from time to time been cursorily noticed in several periodicals, when a circumstantial and elaborate disquisition on its merits and construction appeared in the *Edinburgh Review* for July 1834. It was from perusal of this article that Scheutz, at that time the editor of a technological journal in Stockholm, derived the first conception of constructing a machine for effecting the same purpose as that of Babbage, namely of calculating and simultaneously printing numerical tables. But after he had satisfied himself of the practicability of the scheme, by constructing various models, composed of wood, pasteboard, and wire, he postponed to a future period the further prosecution of the design (No. 10). Three years afterwards, in the summer of 1837, his son EDVARD (1821–1881), then a student at the Royal Technological Institute, Stockholm, took up the problem of constructing a working model in metal. Working with his father, many improvements in the original conception were introduced, and numerous alterations effected. By 1840 the apparatus was so far completed that it correctly calculated the value of series to 5D and one difference also of five figures. By April 1842 the model was extended so as to calculate similar series with two and three orders of differences. In 1843 the printing apparatus and all other parts of the model were in readiness for the inspection of the Royal Swedish Academy of Sciences. After several trials a certificate of the machine's performance was given in Sept. 1843. Making use of this certificate as a recommendation the inventors sought for orders in various countries; but meeting with no success, the model was shut up in its case during the ensuing seven years (No. 10).

In 1850 another inspection was made by a Committee of the Royal Academy of Sweden, and in 1851 Georg Scheutz made an application to the Government for the means to construct a large and still more improved machine. Such a grant was finally made. The new machine, Difference Engine no. 1, was completed in October 1853, being manufactured by C. W. Bergstrom of Stockholm. From the first it was found to work perfectly. During the latter part of 1854 the inventors visited England and France. The machine aroused considerable interest among a number of men of science. Upon being placed in the Great Exhibition at Paris, in 1855, the jury awarded a gold medal to the inventors. In February 1856 Georg Scheutz was made a member of the Swedish Academy of Sciences and in the same year created a Knight of the Order of Vasa and St. Anna. In 1858 he was also

created a Knight of the Order of the North Star. The Academy voted him an annuity of 1200 riksdollar[1] in 1860; and in 1872 the Academy awarded the Carl John Prize, to "Georg Scheutz, the first who successfully clothed Shakespeare in a Swedish costume, and for whom literature, even though as author, an occupation, comprehensive and pursued to the evening of a long life, has in addition for a long period had a connection which has not been effaced by the fact that the man of letters has also acquired a respected name for himself in a field which lies beyond the boundaries of belles lettres." (No. 25).

Through the interest of the astronomer B. A. GOULD, then Director of the Dudley Observatory at Albany, N. Y., JOHN F. RATHBONE,[2] a manufacturer of that city, purchased Difference Engine no. 1 for $5000, and presented it to the Observatory in 1856. There it remained unused until sold in 1924 to the Felt & Tarrant Manufacturing Co., Chicago, Ill., where it may still be seen, in a closed display case at the company's museum at 1735 North Paulina St., Chicago 22.

Scheutz's Difference Engine no. 2, with improvements, was in 1858 constructed for the office of the Register General at Somerset House, by Messrs. Bryan Donkin & Co., from drawings of Edvard Scheutz. It contained some modifications of D. E. no. 1, and was used in the next few years for computations indicated in Nos. 17 and 21 below. In 1914 the machine was presented to the Science Museum, South Kensington where it is still exhibited and occasionally operated for visiting experts.

The descriptions and illustrations in No. 31 are interesting, and the capabilities of the machine are somewhat elaborately set forth in No. 18. Charles Babbage and his son Henry were enthusiastic in their praise of the achievements of the Swedish inventors (Nos. 4, 5, 9). In No. 9 Charles Babbage, who urged upon the Royal Society that their Engine was "highly deserving of a Medal," added

"The principle of Calculation by Differences is common to Mr. Scheutz's engine and to my own, and is so obviously the only principle, at once extensive in its grasp and simple in its mechanical application, that I have little doubt it will be found to have been suggested by more than one antecedent writer.

"Mr. Scheutz's engine consists of two parts,—the Calculating and the Printing; the former being again divided into two,—the Adding and the Carrying parts.

"With respect to the Adding, its structure is entirely different from my own, nor does it even resemble any one of those in my drawings.

"The very ingenious mechanism for carrying the tens is also quite different from my own.

"The Printing part will, on inspection, be pronounced altogether unlike that represented in my drawings; which, it must also be remembered, were entirely unknown to Mr. Scheutz."

Among Edvard Scheutz's own inventions was a rotary steam engine which was used on some steamboats. In 1860 the Swedish Academy awarded him a prize for this. (Nos. 24, 30; see also *Poggendorff's Biographisch-Literarisches Handwörterbuch*, v. 3_2, Leipzig, 1898.)

[1] A riksdollar is worth a little more than an American dollar.
[2] There is a brief biography of JOHN FINLEY RATHBONE in *Appleton's Cyclopædia of American Biography*, New York, v. 5, 1888.

Bibliography

1. Acad. d. Sci., Paris, *C.R.*, v. 7, 1838, p. 1056.

[Statement based on a communication of Georg Scheutz, concerning his calculating machine, announced as being more simple, and consequently less costly than that of Babbage.]

2. *Biographiskt Lexicon* . . ., v. 14, Upsala, 1847, p. 54–66. Supplemented in *Svenskt Biografiskt Lexikon*, v. 9, Stockholm, 1883, p. 495–496. On p. 63–64 of *B.L.* is reprinted a statement about the Scheutz machine in Svenska Vetenskapsakad., Minutes, 1843, Sept. 18.

3. GREAT BRITAIN, *Specifications of Inventions*, v. 83, no. 2216. 11 p. + 2 large sheets (Plates, v. 30) of specifications dated 17 Oct., 1854 ($20\frac{1}{2} \times 28$ ins.) London, 1855.

[Copy in the Boston Public Library; possibly the volume numbers given above are those assigned by the Library in binding. "Letters Patent to Georg Scheutz of Salisbury Street, in the County of Middlesex, Gentleman, and Edward Scheutz, of the same place, Civil Engineer, for the Invention of 'Improvements in Machinery or Apparatus for Calculating, and Printing the Results of Such Calculations.'" The sworn statement of the Scheutzs is concluded on p. 11 and dated 9 March, 1855.]

4. CHARLES BABBAGE, "Note sur la machine Suédoise de MM. Schutz [*sic*] pour calculer les tables mathématiques, par la méthode des différences, et en imprimer les résultats sur des planches stéréotypes," Académie d. Sci., Paris, *C.R.*, v. 41, 1855, p. 557–560, 591.

[A lecture based on charts prepared by his son HENRY PREVOST BABBAGE.]

5. HENRY P. BABBAGE, "On mechanical notation as exemplified in the Swedish calculating machine of Messrs. Scheutz," B.A.A.S., *Report 1855*, "Notices and Abstracts," p. 203–205. Reprinted as a pamphlet (4 p.) with slightly changed title: *Mechanical Notation, exemplified on the Swedish Calculating Machine of Messrs. Scheütz. . . . A paper read at the British Assoc. held at Glasgow, Sept. 1855.* 11 × 17.2 cm.

[There is a copy of this pamphlet in the library of the Harvard School of Business Administration; it is also listed in the catalogue of the Staatsbibliothek, Berlin.]

°6. *Machine à Calculer, qui présente les résultats en les imprimant elle-même. Inventée par Georges Scheutz et Edouard Scheutz.* Stockholm, 1855, 4to. 4 p.

7. "New calculating machine," *Illustrated London News*, v. 26, 30 June, 1855, p. 661.

[There are two illustrations of the machine, and two samples of tables as printed by it. It is noted that the machine was inspected by Prince Albert, at the Royal Society's rooms.]

8. CHARLES MANBY, "Scheutz' Difference Engine and Babbage's Mechanical Notation," Institution of Civil Engineers, *Minutes of Proc.*, v. 15, 1856, p. 497–514; reprinted as a pamphlet, 8A. London, 1856, 8vo, 20 p. [Copy in the British Museum; listed in No. 25.] In *Minutes of Proc.*, v. 16, 1857, p. 224, there is a note of 11 lines telling of the exhibition and sale of the machine.

9. CHARLES BABBAGE, *Observations addressed, at the last anniversary, to the President and Fellows of the Royal Society after the Delivery of the Medals.* London, John Murray, 1856. 12 p.

[Not printed in R. S. London, *Proc.* In the Boston Public Library is a copy with title on a paper cover *On the Swedish Tabulating Machine of Mr. George Scheutz.* On p. 11 are advertisements of Babbage publications.]

10. [GEORGE & EDWARD SCHEUTZ], *Specimens of Tables, Calculated, Stereomoulded, and Printed by Machinery.* London, Longman, . . ., 1856. xviii, 50 p. + frontispiece plate + paper cover.

[This pamphlet was dedicated to CHARLES BABBAGE "by his sincere admirers, George and Edward Scheutz." The Swedish name "Edvard" is here transformed to Edward. On p. 11–42 is the first of the "specimens of tables," log N, N = [1000(1)10000; 5D], calculated, stereomoulded and printed by machinery. On p. 45–50 are 14 short specimens of other tables which might have been similarly elaborated; nos. 2–3 values of polynomials for successive integral values of the variable; nos. 4–5 log N to 7D, with characteristics, for series of successive integers; no. 6 log tan A, A = [27°0′ (1″)27°0′50″; 7D], and similarly, to 7D, nos. 7–8 for log sin A, for A = 1°, and 45°. No. 9 sin A = .230(.001).270, A is given in degrees, minutes, and to the nearest tenth of a second. Nos. 10–11 are a few values of ranges of shot with various charges and log value of male life in London. The last four, p. 49–50, are astronomical, for example, no. 12 is the log radius vector of Venus, and no. 13 the Sun's longitude for every twenty-four hours.

In the Boston Public Library is a copy of the *Specimens* with "Presented to Bowditch Library by Edward Scheutz, 15 Park St., Westminster, May 11th, 1857," on the cover in the handwriting of the donor. In this copy also is a printed slip of paper headed, just before the preface: *Note at the foot of page 7.* The Note is as follows: "In the next machine this rate of working may easily be increased tenfold, so that twenty pages might be quietly *calculated* and *stereomoulded* whilst a compositor was merely '*setting up*' a single page. The addition of fifth and sixth, or indeed of any number of differences, would not occasion any diminution in the rate of working.

"These facts are sufficient to show how vain it would now be to attempt reprints of the existing tables (even if uncertainty and error be disregarded) in any other way than by machinery.

"The object of the Messrs. Scheutz in the present machine was not great speed; and if they had attempted to do more than they actually have done, the reader of the foregoing little history will readily see, they, from mere want of means, would probably have produced no *working* machine at all."

Specimens was reviewed in (i) Institution of Civil Engineers, *Minutes of Proc.*, v. 16, 1857, p. 422; (ii) *The Anthenaeum*, no. 1545, 6 June 1857, p. 720–721; [Possibly written by A. DEMORGAN; quotation: "the tables before us were *stereoglyphed.* We cannot accept such a hybrid as *stereomoulded.*"]; °(iii) *The Practical Mechanics Jn.*, v. 10, 1857, p. 78; °(iv) *Daily News*, London, no. 4324, Mar. 22, 1857. Nos. (iii) and (iv) are listed in No. 25.

It was the recent acquisition of a copy of No. 10 by the Library of Brown University, coupled with the observation of various misstatements of fact about the Difference Engine, which led to the compilation of the present article, with a rather complete Bibliography, by means of which the interested reader may still further enlarge his knowledge in this regard.]

11. *Specimen de Tables calculées, stéréotypées et imprimées au moyen d'une Machine.* Paris, 1858. 8 vo. 68 p. + 1 plate.

[Also dedicated to CHARLES BABBAGE. Reviewed in °(i) *Propriété Industrielle*, Paris, 1858, no. 34; °(ii) *Siècle*, Paris, 1858, no. 8533; °(iii) *Le Pays*, Paris, Sept. 24, 1858. (i)–(iii) are listed in No. 25.]

12. A. RIVIÈRE, "Machine à calculer," *L'Illustration, Journal Universel*, Paris, v. 32, 28 Aug. 1858, p. 143, illustrated by a picture of the machine.

13. F. N. M. MOIGNO, *Cosmos*, v. 13, 1858, p. 78–84.

[An account, with a full-page illustration, of the Scheutz machine, and reports, with comments, of discussions by Babinet, Leverrier, and others, at a meeting of the Académie des Sciences, Paris, 12 July 1858.]

°14. Three articles in *Aftonbladet* [Evening News], 1859, nos. 14, 17, 253. [Listed in No. 25.]

°15. *Report on the Calculating Machine recently constructed by Mr. Donkin.* London, 1859. 4to. 4 p. [Listed in No. 25. An account of Difference Engine no. 2.]

16. G. SCHEUTZ, "Scheutz's Räknemaskin," Svenska Vetenskapsakad., Stockholm, *Ofversigt af . . . Förhandlingar*, v. 16, 1859, p. 391.

°17. *Mountain Barometer Tables: Calculated and Stereoglyphed* by Messrs. Scheutz's Calculating *Machine no. 2*, and *printed by machinery*, London, 1859. 19 p.

[A small booklet, 2 × 6½ ins., "for private circulation," printed for WM. GRAVATT F.R.S., foreign member of the Royal Academy of Sweden—the first publication of results from Difference Engine no. 2. There has been a copy in the Science Museum, South Kensington, since 1914. Listed in Nos. 25 (with date 1859) and 31 (with date 1862). The title here seems to have been influenced by the criticism in No. 10 (ii).]

18. H. MEIDLINGER, "Die Scheutz'sche Rechenmaschine," *Dinglers Polytechnisches Jn.*, Stuttgart and Augsburg, s. 4, v. 6, 1860, p. 241–256, 321–336.

19. A. DEMORGAN, "Table," *English Cyclopaedia, Arts and Science Section*, London, v. 7, 1861, col. 1007, 9 lines.

[The date of publication of No. 17 is here given as 1859.]

20. CHARLES BABBAGE, *Passages from the Life of a Philosopher*, London, 1864, p. 47–48.

21. *English Life Tables. Tables of Lifetimes, Annuities, and Premiums. With an Introduction by William Farr, M.D., F.R.S., D.C.L. Published by Authority of the Registrar General of Births, Deaths, and Marriages in England.* London, printed for Her Majesty's Stationery Office, 1864. clvi, 606 p. "English Life Table no. 3" p. 6–605.

[Appendix, "Scheutz's calculating machine and its use in the construction of the English Life Table no. 3," p. cxxxix–cxliv. "This volume is the result; and thus—if I may use the expression—the soul of the machine is exhibited in a series of Tables which are submitted to the criticism of the consummate judges of this kind of work in England and in the world," p. cxl. The tables calculated and printed by Difference Engine no. 2 are on p. 6–11, 42–47, 76–81, 114–115, 129–133, 142–145, 175–245 (mostly), 355–425 (mostly), 481–551 (mostly). There are copies of these tables at Harvard University, and in the South Kensington Museum.]

22. "The Swedish tabulating machine of G. & E. Scheutz," *Annals of the Dudley Observatory*, v. 1, Albany, 1866, p. 116–126 + large folding plate of the machine on a stand.

23. J. W. L. GLAISHER, "Report of the Committee on mathematical tables," reprint from B.A.A.S., *Report 1873*, p. 59, 159. 6 lines about No. 10.

24. HERMAN HOFBERG, *Svensk Biografiskt Handlexicon*, Stockholm, v. 2, 1876, p. 242–243.

[Sketches of P. G. and E. Scheutz. Copy at Library of Congress.]

New edition, Stockholm, v. 2, 1906, p. 428–429:

[Biographical sketches, with portraits, of Per Georg Scheutz and Edvard Scheutz. Copy at Harvard University. This, No. 25, No. 2, and No. 10, were the main bases for the compilation of our sketch.]

25. C. F. BERGSTEDT, "Georg Scheutz," Svenska Vetenskapsakad., *Lefnadsteckningar*, v. 2, part 1, Stockholm, 1878, p. 155–179.

[The most extensive biography of Scheutz, with a complete bibliography of his publications.]

26. MAURICE D'OCAGNE, *Le Calcul Simplifié par les Procédés Mécaniques et Graphiques*, Paris, 1894, p. 37–38.

Second ed., Paris, 1905, p. 83–86, 213–218.

Third ed., Paris, 1928, p. 74–77, 182–187 ("Note sur la machine à différences Scheutz" by Colonel BERTRAND).

[In the third edition, there is a picture of the machine on p. 75. On p. 76 it is incorrectly stated that after Difference Engine no. 1 was at Albany it was utilized for the calculation of tables of logarithms, sines, and logarithmic sines (see No. 9). The statement concerning Difference Engine no. 2, "Cette machine a calculé et imprimé 605 tables (grand in-4°) qui constituent le fondement du calcul des rentes viagères servies par les caisses d'épargne postale anglaises" is also highly inaccurate and misleading; see No. 21.]

27. V. G. VON BOOL, *Pribory i mashiny dlíâ mekhanicheskago Proizvodstva arifmeticheskikh Deĭstviĭ* [Apparatuses and machines for mechanical execution of arithmetical operations]. Moscow, 1896, p. 184–189.

[There is a copy of this work in Library of Congress, by VLADIMIR GEORGÏEVICH VON BOOL (1835–1889). A picture of the Scheutz Difference Engine no. 1 fills p. 189].

28. R. MEHMKE & M. D'OCAGNE, "Calculs numériques," *Encycl. d. Sci. Math.*, I, 4. 2. Paris, 1908, p. 268–269.

[Mostly accurate and with good bibliography; there are a number of errors in the reference to no. 27.]

29. L. JACOB, *Le Calcul Mécanique*, Paris, 1911, p. 115–117. [Seems to be an abridgment of No. 26, second ed.]

30. *Kungl. Svenska Vetenskapsak., Personforteckningar, 1739–1915*, ed. E. W. DAHLGREN, Stockholm, 1915.

31. D. BAXANDALL, *Mathematics. I. Calculating Machines and Instruments* (*Catalogue of the Collections in The Science Museum, South Kensington with Descriptive and Historical Notes and Illustrations.*) London, 1926, p. 32, 34–36 plate IX (i. Scheutz's Difference Engine no. 2; ii. Details of wheelwork; iii. Portion of a calculated and printed table).

[The entries of the *Catalogue* are: no. **86**, portrait of Edvard Scheutz during his student days at the Royal Technological Institute, Stockholm. No. **87**, no. 7. No. **88**, photograph of the Scheutz Difference Engine no. 1 at the Dudley Observatory. No. **89**, No. 10. No. **90**, Scheutz Difference Engine no. 2, presented by the General Register Office, Somerset House, 1914. No. **91**, No. 17, but with date 1862. No. **92**, No. 21. No. **93**, moulds and stereotypes produced by the Scheutz Difference Engine no. 2. No. **94**, Stereotype block from a matrix produced by the Scheutz Difference Engine no. 2. No. **95**, Lead mould, papier-mâché moulds (3) and stereotype produced by the Scheutz Difference Engine no. 2.]

32. *Svensk Uppslagsbok*, v. 24, Malmö, 1937.

[Sketch and portrait of P. G. Scheutz, col. 119].

33. H. H. AIKEN, etc., *A Manual of Operations for the Automatic Sequence Controlled Calculator* (*Annals* of the Computation Laboratory of Harvard University, v. 1). Cambridge, Mass., 1946, p. 6–7.

[Quotation: "In 1834 George Scheutz, a printer in Stockholm, built a less ambitious difference engine with the aid of a grant from the Swedish government. This machine was completed in 1853 and used for the computation and printing of tables of logarithms, sines, and logarithms of sines." In view of the details given above the first sentence clearly conveys more than one erroneous impression. The latter sentence is equally misleading since only

the tables described in No. 10 are in question. Because the reviewer in *MTAC*, v. 2, p. 186, followed the Harvard *Annals* v. 1, he listed "SCHEUTZ (1834)", rather than SCHEUTZ (1853).]

R. C. A.

A New Approximation to π

A. EDITORIAL NOTES: In *MTAC*, v. 2, p. 143–145 we noted various formulae which had been used for calculating π to many places of decimals. These included that of MACHIN (1706)

$$\tfrac{\pi}{4} = 4 \tan^{-1} \tfrac{1}{5} - \tan^{-1} \tfrac{1}{239}, \tag{1}$$

which was used by WILLIAM SHANKS (1812–1882) to compute π to 707D. The accuracy of this computation to 500D was verified by an independent calculation completed and published in 1854. No one appears to have checked the later figures until 1945, when Mr. D. F. FERGUSON, now connected with the Department of Mathematics of the University of Manchester, undertook the task. As we have already noted he used the formula

$$\tfrac{\pi}{4} = 3 \tan^{-1} \tfrac{1}{4} + \tan^{-1} \tfrac{1}{20} + \tan^{-1} \tfrac{1}{1985}, \tag{2}$$

given in LONEY's *Plane Trigonometry* (1893). We published his new computation for π from the 526th through the 620th decimal place (p. 145). Mr. Ferguson gave an account of his work in *Mathematical Gazette*, v. 30, May 1946, p. 89–90, and recorded there his figures for π from the 521st to the 540th decimal place. Mr. Ferguson found that Shanks' approximation to π was incorrect beyond 527D. By November 1946 he had carried on his calculations of the value of π to 700D, and by January 1947 to 710D.

In December 1945 we suggested to Dr. JOHN W. WRENCH, JR. that he might take up the wholly independent computation of π by means of Machin's formula (1). In April 1946 he reported that he was in communication with Mr. LEVI B. SMITH of Talbotton, Georgia, who began his work on computing $\tan^{-1} \frac{1}{239}$ in November 1940 and had by February 1944 completed the work to 820D, through the term $[173 \cdot 239^{173}]^{-1}$. Then Dr. W. took up actively the computation of $\tan^{-1} \frac{1}{5}$ so that his results might be combined with those of Mr. S. as in Machin's formula. He found the errors in work of Shanks, earlier pointed out by Mr. F., and others described below.

Early in January 1947 Dr. W. sent to us his new approximation to π to 808D given below, as a companion to the value of e to 808D (*MTAC*, v. 2, April 1946, p. 69). The value found by Mr. F. to 710D agrees with this.

π = 3.14159	26535	89793	23846	26433	83279	50288	41971	69399	37510
58209	74944	59230	78164	06286	20899	86280	34825	34211	70679
82148	08651	32823	06647	09384	46095	50582	23172	53594	08128
48111	74502	84102	70193	85211	05559	64462	29489	54930	38196
44288	10975	66593	34461	28475	64823	37867	83165	27120	19091
45648	56692	34603	48610	45432	66482	13393	60726	02491	41273
72458	70066	06315	58817	48815	20920	96282	92540	91715	36436
78925	90360	01133	05305	48820	46652	13841	46951	94151	16094
33057	27036	57595	91953	09218	61173	81932	61179	31051	18548
07446	23799	62749	56735	18857	52724	89122	79381	83011	94912
98336	73362	44065	66430	86021	39494	63952	24737	19070	21798
60943	70277	05392	17176	29317	67523	84674	81846	76694	05132
00056	81271	45263	56082	77857	71342	75778	96091	73637	17872
14684	40901	22495	34301	46549	58537	10507	92279	68925	89235
42019	95611	21290	21960	86355	44191	19716	02977	46113	09960
51870	72113	49999	99837	29780	49951	05973	17328	16096	31867
50244	594(55)								

B. Report of Mr. Smith & Dr. Wrench, January 1947

As stated in the Editorial Notes formula (1) was employed by the present writers in their joint calculation of π. The calculation of $\tan^{-1} \frac{1}{5}$ consisted essentially of checking and extending to 850D the values of the individual terms of the corresponding series as published to 530D by Shanks.[1] On the other hand, the computation of $\tan^{-1} \frac{1}{239}$ was carried out to 820D entirely independently of earlier calculations of that number, and the resulting values of the terms of the series were checked against the corresponding data of Shanks only when the investigation was nearly completed.

An important preliminary step in the new evaluation of $\tan^{-1} \frac{1}{5}$ consisted of the formation of a definitive table of powers of 2. This original table contains the exact values of 2^n for $n = 1(2)1207$, and has been collated with an unpublished table of non-consecutive powers to 2^{671} computed by Professor H. S. Uhler[2] and also with a table of 2^n, $n = 13(12)721$, given by Shanks.[1] No discrepancies were found. In addition to this comparison with previous tables of high powers of 2, every entry beyond 2^{531} in the new table was checked by the Fermat-Euler theorem.

The quotient arising from the application of this congruential check on the accuracy of the tabular value of 2^{2n-1} comprised, together with the appropriate number of antecedent zeros, the sequence of digits occupying the first $2n - 1$ decimal places of the approximation to the nth term of the series for $\tan^{-1} \frac{1}{5}$. If r denotes the residue determined by the preceding check, then the decimal evaluation of $r/(2n - 1)$ is also required, corresponding to all integral n between 1 and 425. Thus it was found desirable to calculate *de novo* a table of the complete periods of the reciprocals of all prime-powers (p^k, $k \geqq 1$, $p \neq 2, 5$) less than 800. In each of the many cases where the period consisted of an even number of digits an effective check involved the juxtaposition and subsequent addition of the two halves of the period so as to yield an unbroken sequence of 9's.[3] All remaining cases of evaluation of $r/(2n - 1)$ were checked by duplicate machine calculation, as was the summation of the terms of the series.

The calculation of the successive terms of the series for $\tan^{-1} \frac{1}{239}$ was performed by the recurrence formula

$$U_{n+1} = (2n - 1)U_n/(2n + 1)239^2,$$

where U_n and U_{n+1} denote respectively the nth term and its successor. The computations were carried to at least 820D and were checked modulo $10^{10} + 1$ every hundred decimal places. The same checking procedure was applied to the respective sums of the positive and negative terms of the series.

The final values of $\tan^{-1} \frac{1}{5}$ and $\tan^{-1} \frac{1}{239}$ were compared with the corresponding data to 709D of Shanks, and several errata in the latter were discovered. In addition to the two errors (described in **C**) which were independently discovered by Mr. Ferguson in Shanks' value of $\tan^{-1} \frac{1}{5}$, there exist in the same number a unit error in the 533rd place and a residual error of approximately $5.2193762669 \times 10^{-602}$. Shanks' approximation to $\tan^{-1} \frac{1}{239}$ is also erroneous, for it exceeds the present estimate of that number by nearly $4.77447473 \times 10^{-592}$.

Appended to this report are values of (**a**) $\tan^{-1} \frac{1}{5}$, and (**b**) $\tan^{-1} \frac{1}{239}$, both curtailed to 811D from more extended approximations appearing on the work sheets.

(**a**)

0.19739	55598	49880	75837	00497	65194	79029	34475	85103	78785
21015	17688	94024	10339	69978	24378	57326	97828	03728	80441
12628	11807	36913	60104	45647	98867	94239	35574	75654	95216
30327	00522	10747	00156	45015	56006	12861	85526	63325	73186
92806	64389	68061	89528	40582	59311	24251	61329	73139	93397
11323	35378	21796	08417	66483	10525	47303	96657	25650	48887
81553	09384	29057	93116	95934	19285	18063	64919	69751	94017
08560	94952	73686	73738	50840	08123	67856	15800	93298	22514
02324	66755	49211	02670	45743	78815	47483	90799	78985	02007
52236	96837	96139	22783	54193	25572	23284	13846	47744	13529
09705	46512	24383	02697	56051	83775	74220	87783	58531	52464
74933	09145	87633	82311	24903	32030	12680	51006	70223	31257
50509	42448	46026	71622	54894	07922	61404	67995	06236	59692
82873	05828	78720	53603	03457	07660	66681	37431	25662	67431
40899	26057	41703	54539	40465	13623	01101	58081	00262	13759
92595	89071	66648	51452	55706	79954	88100	43132	95466	83892
79036	88309	3							

(**b**)

0.00418	40760	02074	72386	45382	14959	28545	27410	48065	30763
19508	27019	61288	71817	78341	42289	32737	82605	81362	29094
54975	45066	64448	63756	05245	83947	89311	86505	89221	28833
09280	08462	71962	33077	33759	47634	60331	84734	14570	33198
60154	54814	80599	24498	30211	46039	12539	49527	60779	68815
58881	27339	78533	46518	04574	25481	35867	46447	51979	10232
83097	70020	64652	82763	46532	96910	48183	86543	56078	91959
14512	32220	94463	68627	66155	20831	67964	26465	74655	11032
51034	35262	82445	12693	55670	49968	44452	47904	33177	28393
07086	31401	93869	51950	37058	64107	70855	85540	45223	55388
14237	67708	36515	69182	52702	00229	30895	44950	04358	54409
34496	44014	24187	24950	92283	86239	54553	33565	11719	73747
02023	49475	97790	97469	50111	88854	76673	97957	31537	09303
27821	13089	84258	30836	77190	91008	39098	51655	10419	22416
78092	05326	86491	62667	40271	68444	24477	31579	64520	27549
57415	88258	29094	05850	90382	07331	75908	43199	77843	27604
28586	38373	5							

[1] W. SHANKS, *Contributions to Mathematics comprising chiefly the Rectification of the Circle to 607 Places of Decimals*, London, 1853.

[2] *MTAC*, v. 2, p. 224, N66.

[3] H. RADEMACHER & O. TOEPLITZ, *Von Zahlen und Figuren*, second ed. Berlin, 1933, p. 133–135.

C. REPORT OF MR. FERGUSON, January 1947

In calculating my value of π to 710D I made use of formula (2) and found the following results for (**a**) $\tan^{-1} \frac{1}{4}$, (**b**) $\tan^{-1} \frac{1}{20}$, and (**c**) $\tan^{-1} \frac{1}{1985}$:

(**a**)

0.24497	86631	26864	15417	20824	81211	27581	09141	44098	38118
40671	27375	91466	73551	19587	64209	65745	34157	66870	19913
63834	80449	00371	18374	29548	54209	95059	97695	89869	60614
20373	52012	77087	38758	16557	21586	71598	26385	50632	05220
87873	06750	14341	56233	63482	63956	36978	08521	59107	32458
35238	13507	62999	55568	90112	58302	66262	33025	99157	53281
02760	62335	60275	36107	52021	78574	13846	85151	60692	64028
13351	40849	79441	00602	37988	46394	26115	26552	60206	13960
37795	44423	80726	04741	94391	78861	19777	28818	69907	30107
50590	15074	22498	58439	41420	59686	03414	25179	53473	22290
10855	05491	71108	69928	36288	80499	01695	01187	90130	24132
23900	98621	56045	96838	55524	50318	46160	29411	55558	05517
39341	51460	87605	90907	76086	95391	27283	64199	28418	54722
42486	09165	39440	71147	52423	62989	80883	85199	09565	57220
81885	50930	(59)							

(b)

0.04995	83957	21942	76141	00062	87034	84488	14912	77080	42350
71744	10853	45482	99835	95476	71033	50612	64888	70485	01265
49675	88718	56799	74803	45043	78235	17343	64195	86075	35558
34705	50031	66812	64425	55070	35889	99864	21844	62020	22011
35398	44491	94479	55125	91884	70605	15358	82203	57911	15507
66709	70265	20884	04697	53559	08904	34425	33211	75071	00898
99983	99369	89611	53196	70717	40134	40774	24235	31335	37603
73612	47259	31779	72222	52596	59464	82850	02739	09656	29682
61838	28530	42311	66214	89812	84597	81323	80425	73403	65277
35640	82643	15372	91283	73850	56089	49548	56557	20164	84879
81610	72192	83012	94406	89240	40051	11637	64820	56557	65999
47240	24101	35373	55511	99718	11544	33853	54021	44594	33781
36222	03768	16540	61055	38956	20032	50668	29159	05403	66710
93525	58744	35937	48968	67734	87127	37233	28015	36651	95672
97735	23477	(67)							

(c)

0.00050	37782	94913	08568	94071	15151	20340	68155	82974	27671
70794	50754	94924	49051	47331	91562	48721	41344	62457	31663
54356	72097	25292	46735	43656	70658	85121	98509	57714	34631
16201	87555	72950	86848	41560	75739	76456	56371	24816	21875
12054	38001	54144	09788	49786	28731	58173	88022	96546	91890
13988	03384	98768	15748	37461	32808	20136	07891	78079	24576
34848	81139	81141	28185	10421	04373	41755	93445	09515	54421
06064	77781	30180	52296	70643	13015	42264	54341	08263	07459
83039	69957	29909	17547	54316	44112	04827	48412	99637	24037
14450	28084	07818	72581	81602	03033	62489	37749	65168	46978
10408	29672	64677	37415	73398	53325	49265	37800	02819	17053
46292	72603	22836	58266	41037	79381	23834	28205	57905	00949
45767	62167	06957	55242	02247	36629	36425	52266	02749	98589
82687	23984	76364	21164	11631	63537	47742	14457	26882	79973
42113	22633	(35)							

$(b) + (c) = \tan^{-1}(5/99)$, which was independently computed and furnished complete agreement to 710D+.

My procedure for calculating $\tan^{-1}\frac{1}{4}$ was as follows:

(i). calculated $(1/4)^{2n+1}$ by dividing $(1/4)^{2n-1}$ by 16;
(ii). multiplied the result by 4096 and compared with $(1/4)^{2n-5}$;
(iii). divided $(1/4)^{2n+1}$ by $(2n + 1)$;
(iv). multiplied this by $16(2n + 1)$ and compared with $(1/4)^{2n-1}$;
(v). after copying $(1/4)^{2n+1}/(2n + 1)$ for the purpose of the series, checked by multiplying the copied figures by $2n + 1$.

At all steps of the work ample margins of overlap were allowed.

In the course of my work I discovered two errors in the results of Shanks. The explanation of the first of these which vitiates his final result beyond 527D was noted in January 1946 and is as follows: It was a question of an omission in the evaluation of the term $[497 \cdot 5^{497}]^{-1}$ in the 531st decimal place. I found the value to be (through 547D)

00804 82897 38430 58350,

while Shanks, carelessly omitting a zero, used

00848 28973 84305 83501

The second Shanks' error was the omission from 569D+ of the term $5^{-29}/29$ which comes in the series for $\tan^{-1}\frac{1}{5}$.

RECENT MATHEMATICAL TABLES

359[A].—A. L. CRELLE, *Rechentafeln welche alles Multiplizieren und Dividieren mit Zahlen unter Tausend ganz ersparen, bei grösseren Zahlen aber die Rechnung erleichtern und sicherer machen. Neue Ausgabe besorgt von O. Seeliger* [1907]. *Neudruck, mit Tafeln der Quadrat- und Kubikzahlen von 1–1000.* Berlin, Gruyter, 1944. viii, 501 p. 24.8 × 36.7 cm. See also *MTAC*, v. 2, p. 179.

A. L. CRELLE (1780–1855) was the founder (1826) of a notable mathematical research journal still in existence, and author of various volumes, including even one on music (1823). His calculating tables, of which there have been many editions in English, French, and German, was first published in two small volumes over 125 years ago, Berlin, 1820. But in such editions, since 1857 at least, the folio format (each page containing four pages of the first edition) has been in use up to the present. As the title indicates the war edition under review contains two extra pages with the squares and cubes of numbers N, $N = 1(1)999$. A one-volume Japanese edition of Crelle's tables by TSUNETA YANO, Tokyo, 1913, has been referred to, *MTAC*, v. 2, p. 18; our incidental note, v. 1, p. 436, that this was published by an insurance company, is incorrect. This fact was learned after Mr. EDWIN G. BEAL, Jr., Chief of the Japanese Division in Library of Congress had kindly made a study of the volume for us. He also reported that he could find no record of a 1927 Japanese edition of this work.

R. C. A.

360[A, B, F].—J. SER, *La Numération et le Calcul des Nombres*, Paris, Gauthier-Villars, 1944, 194 p., 25 × 16 cm.

The author of this work appears to be something of an individualist. He gives but one reference to the work of others and this is to an article on the foundations of mathematics, a subject not covered in this book. The reader will find many new points of view, "méthodes personelles," and unfamiliar nomenclature. Much of this has to do with pencil and paper calculation, a discipline all but unknown in this age of mechanized computation.

The more extensive tables in this work may be described as follows:

(i) An arithmetical table (p. 60–95) giving for the first 1000 integers N the following functions:

$1/N$ to 6D, the first nine multiples of N (these are used to facilitate multiplication and division), N^2, $\sqrt{N}$ to 4D, $\sqrt{10N}$ to 4D, N^3, $N^{\frac13}$ to 4D, $(10N)^{\frac13}$ to 3D, and $(100N)^{\frac13}$ to 3D.

(ii) A small table, p. 96, of powers n^k for $n = 1(1)9$, $k = 1(1)16$, the first nine multiples of $10^{\frac12}$, $10^{\frac13}$, $100^{\frac13}$ (all but five values have last-digit errata) and the fourth roots of $m\cdot10^k$, $m = 1(1)9$, $k = 1, 2, 3$ to 4D.

(iii) A table of the roots x of the linear congruence

$$Rx \equiv D \pmod{B}$$

where R and B range over the first 50 integers and D is the greatest common divisor of R and B (p. 100–101). The reader will find that values of R are given as column headings.

(iv) A table of the residues with respect to each of the moduli 2, 3, 5, and 7 of the first 210 integers arranged in two ways (p. 122–123).

(v) A factor table for each of the first 1000 integers except multiples of 10. To save space composite numbers are usually broken into only two factors, one of which is often a composite number (p. 144–145).

(vi) A factor table for those numbers between 1000 and 10000 which are prime to 30. Beyond 4020 only the least prime factor is given if the number is composite (p. 146–149).

(vii) A condensed factor table for numbers under 210000, not divisible by 2, 3, 5, or 7. This 12-page table (p. 150–161) is reminiscent of an unfinished project of E. LEBON,[1] and is too complicated to describe here in detail. The reader is warned to study directions before

attempting to use the table. For composite numbers whose least factor exceeds 210, the table yields the two factors at one "coup d'oeil," that is, after a little hunting. For other numbers some mental calculation involving the factoring of 10 or sometimes 25 three-digit numbers is necessary.

D. H. L.

[1] E. LEBON, *Table de Caractéristiques de Base 30 030 donnant, en un seul Coup d'Oeil, les Facteurs Premiers des Nombres Premiers avec 30 030 et Inférieurs à 901 800 900*, v. 1, pt. 1, Paris, 1920.

361[B].—ALBERT GLODEN, *Table des Bicarrés X^4 pour $1000 < X \leqslant 3000$*, Luxembourg, author, rue Jean Jaurès 11, 1946. Offset printing on one side of each of 17 leaves, with paper cover. 20.5 × 29.7 cm.

There is no text. The values of X^4 for $X = 1001(1)1099$ check with the values given in BAASMTC, *Mathematical Tables*, v. 9, 1940, p. 122–123. Some of the printing is unclear so that a "3," one case tested, $X = 1003$, might easily be mistaken for a "5." This table was made preparatory to writing the paper reviewed in RMT **348**.

R. C. A.

362[D].—JOSEF KŘOVÁK, *Natürliche Zahlen der Funktion Cotangens für Winkel in Zentesimalteilung von 0^{gr} bis 100^{gr}*. Prague, Landesvermessungsamt Böhmen und Mähren, second ed., 1943. iii, 396 p. 15.5 × 21.5 cm.

The one-page explanation (dated Prague, 1943) of these tables says that they had already been announced in the *Sechsstellige Tafeln der natürlichen Werte der Funktionen Sinus und Cosinus für Winkel in Zentesimalteilung* that the Finance Minister had published in the previous year.

The tables were brought into being for use with trigonometrical survey calculations with twin calculating machines where the accuracy of measurement is of the order of 2'' (0''.6) or slightly better. In other words, it provides six significant figure values for working to $0^g.0001$ or 1'' or one centesimal second or about one third of a sexagesimal second. They are a further outcome of Hitler's decree that German surveyors were to use the centesimal division of the quadrant.

The principal survey problem that is facilitated by cotangents and twin machines is that of intersection, i.e. the determination of the co-ordinates of a point whose bearings from two known points have been measured.

The lay-out is shown by the following table:

Pages	From	To	Interval	Diff. for 1''
1– 20	0^g	1^g	1''	None
21–160	1	15	2	63.6 to 1.5
161–230	15	50	10	2.9 to 0.3
231–316	50	93	10	3.2 to 1.5
317–376	93	99	2	1.5 to 15.9
377–396	99	100	1	None

Where the interval is 2'', mean differences are given for 1''; where the interval is 10'', complete proportional parts for centesimal seconds are given.

The tables have been typed and reproduced from photographic plates. The Bremiker division of the lines has been used throughout. No description of the source of the values is given.

It seems hard to justify the use of six significant figures throughout if the accuracy of measurement is limited to about 1''. For small angles, the last one, two or even three figures are meaningless. The same is true of angles near 100^g, where the number of decimals increases steadily to 11. These extra decimals cannot be of any use in survey work, and are only likely to be a source of confusion.

It is a blemish on the arrangement of the part of the table that is at interval 10'' (156

pages) that the centesimal minutes 0 to 50 are on a *right-hand* page and 50 to 100 on the following left-hand page; they should, of course, have been printed at a single opening.

L. J. C.

363[D, E, L].—MIKLÓS IMRE HETÉNYI, *Beams on Elastic Foundation. Theory with Applications in the Fields of Civil and Mechanical Engineering.* (*Univ. Michigan Studies. Scientific Series*, v. 16). Ann Arbor, Univ. of Michigan Press, 1946, p. 217–255. 17 × 25.2 cm. $4.50.

The tables include (p. 217–239) graphs and 4D tables $A_x = e^{-x}(\cos x + \sin x)$, $B_x = e^{-x}\sin x$, $C_x = e^{-x}(\cos x - \sin x)$, $D_x = e^{-x}\cos x$, for $x = 0(.001).02(.01)4(.1)8$, $\frac{1}{2}\pi(\frac{1}{4}\pi)\frac{5}{2}\pi$. There are also 5–7S tables with graphs (p. 241–243) of $E_1 = \frac{1}{2}e^x(\sinh x + \sin x)^{-1}$, $F_1 = \frac{1}{2}e^x(\cosh x + \cos x)^{-1}$, $E_{11} = \frac{1}{2}e^x(\sinh x - \sin x)^{-1}$, $F_{11} = \frac{1}{2}e^x(\cosh x - \cos x)^{-1}$, for $x = 0(.05)3(.1)5$.

There are also graphs and 4D tables (p. 245–255) of $Z_1(x) = \operatorname{ber} x$, $Z_2(x) = -\operatorname{bei} x$, $Z_1'(x)$, $Z_2'(x)$, $Z_3(x)$, $Z_4(x)$, $Z_3'(x)$, $Z_4'(x)$ for $x = 0(.01)6$, where

$$Z_3 = \tfrac{1}{2}\operatorname{ber} x - (2/\pi)[R_1 - \operatorname{bei} x(\gamma + \ln \tfrac{1}{2}x)],$$
$$Z_4 = -\tfrac{1}{2}\operatorname{bei} x + (2/\pi)[R_2 + \operatorname{ber} x(\gamma + \ln \tfrac{1}{2}x)], \text{ and}$$
$$R_1 = (\tfrac{1}{2}x)^2 - \frac{\phi(3)}{3!^2}(\tfrac{1}{2}x)^6 + \frac{\phi(5)}{5!^2}(\tfrac{1}{2}x)^{10} - \cdots,$$
$$R_2 = \frac{\phi(2)}{2!^2}(\tfrac{1}{2}x)^4 - \frac{\phi(4)}{4!^2}(\tfrac{1}{2}x)^8 + \frac{\phi(6)}{6!^2}(\tfrac{1}{2}x)^{12} - \cdots, \quad \phi(n) = \sum_1^n 1/k,$$
$$\gamma = .577216\cdots.$$

364[D, P].—ISTITUTO GEOGRAFICO MILITARE, Florence, *Tavole per Calcolare le Differenze di Livello nelle Levate Topografiche e per Calcolare le Distanze ridotte all'Orizzònte.* (*Collezione di Testi Tecnici*). Florence, 1943, viii, 195 p. 19.3 × 23.6 cm. Full cloth. The Preface is signed by Prof. GIOVANNI BOAGA, geodetic chief.

L is the distance AB of an object, α the angle of its elevation or depression with reference to the horizontal plane, $D = AC$ the projection of L on this plane, $L' = AE$ the projection of $AC = D$, on L, and $BC = h$.

Table I, p. 3: $D = L\cos\alpha$ to 4D, for $\alpha = 1°(1°)30°$, $L = 1(1)9$.
Table II, p. 14–41: $L' = D\cos\alpha = L\cos^2\alpha$, to 4 or 5S, for $\alpha = 0(5')5°(2')11°(1')19°59'$ and $L = 1(1)9$; also for $L = 1$, $\alpha = 20°(1')45°$.
Table III, p. 43–135: $h = D\tan\alpha$, to 5D, for $D = 1(1)9$, $\alpha = 0(15'')15°(30'')20°(1')45°20'$.
Table IV, p. 191: corrections due to sphericity and refraction, differences of level in meters 1000(100)25900, coefficient of refraction = .06733.
Table V, p. 195: $\tan^2\alpha$, for $\alpha = 0(50')30°50'$, Tables for correction of sphericity and refraction.

The previous edition of this work appeared in 1915 (15.7 × 22.4 cm., 53 p.) and contained four tables. The first, and last two tables are practically equivalent to the first, fourth, and fifth tables of the 1943 edition. T. II (1915) is of the same plan as T. III (1943), but for the range [0(1')45°; 4D]. This 1915 edition was an enlarged and corrected edition of a previously revised and corrected edition, which appeared in 1896.

R. C. A.

365[D, S].—LOUIS COUFFIGNAL, *Tables de Produits de Lignes Trigonométriques.* Paris, Gauthier-Villars, 1943. iii p. + 24 thick paper leaves, printed on only one side. 31.3 × 23.5 cm. Boards, 210 francs.

This volume was prepared under the direction of Dr. COUFFIGNAL, the director of the laboratory of mechanical calculation in the Centre National de la Recherche Scientifique.

The author's volume, *Les Machines à Calculer. Leur Principes. Leur Evolutions.* (Paris, 1933, ix, 86 p.) is well known. His doctoral dissertation at Paris was entitled *Sur l'Analyse Mécanique. Application aux Machines à Calculer et aux Calculs de la Mécanique Céleste* (Paris, 1938, 132, 3 p. 4to).

On the back of the title-page of the present volume is a brief preface in French, German and English, and on the opposite page, again in three languages, are "Directions for use of the Tables." We are told that "The establishment of crystal structure from X-ray diagrams demands extensive calculations, where products of two or three cosines occur continually. On the request of several French crystallographers the French National Office of Scientific Research has undertaken to publish tables which might facilitate this kind of work. Besides, such Tables may be useful in a great many cases of harmonic analysis."

The tables give the products $P = f(X)\cdot g(Y)\cdot h(Z)$, where f, g, h are either sine or cosine functions. The arguments X, Y, Z are at interval one hundredth of a circumference, that is, $4^g = 3°.6$. From the table one may read off at once the value of P for any X, Y, Z in 4^g units up to 100^g. For example, to evaluate $P = \sin 41\cdot\cos 65\cdot\cos 7$ first turn to $\cos Z = \cos 7$, p. 7 (in the upper right-hand corner of the page). Then on that page columns $\sin X = \sin 41$, $\cos Y = \cos 65$, indicate that $P = -.2850$. The results are all to 4D. The author states that the error in any P is less than $5\cdot 10^{-5}$.

R. C. A.

366[F].—A. Gloden, "Compléments aux tables de factorisations de Cunningham," *Mathesis*, v. 55, 1946, p. 254–256. 16.2 × 25 cm.

The tables referred to are those in which Cunningham gives[1] (with many incomplete entries) the factors of numbers of the form $x^4 + 1$ for $x < 1000$. The results quoted in this note serve to complete all but 51 entries in this table. Previous addenda by Kraitchik[2] and Beeger[3] are given and have been verified. The new results are by-products of tables of the solutions of the congruence

$$x^4 \equiv -1 \pmod p$$

for $p < 500000$ by Gloden and Delfeld.[4] Those values of x for which $(x^4 + 1)/d$ is a prime between 10^{10} and $25\cdot 10^{10}$ are listed for $d = 1, 2, 17, 34, 41$, and 82. For some reason the author has failed to list 565 for $d = 1$ and 640, 648 for $d = 2$. Six other factorizations are given for $x = 595, 598, 685, 714, 844, 880$. The author has recently given a similar table[5] to Cunningham's for $1000 < x \leqslant 3000$. The present note closes with a table of the factors of $x^8 + 1$ for $x = 37, 41, 50, 52, 63, 82, 85$, and 87.

D. H. L.

[1] A. J. C. Cunningham, *Binomial Factorisations*, v. 1, London, 1923, p. 113–119.
[2] M. Kraitchik, *Recherches sur la Théorie des Nombres*, v. 2, Paris, 1929, p. 116–117.
[3] N. G. W. H. Beeger, *Additions and Corrections to Binomial Factorisations by Cunningham*. Amsterdam, 1933, 1945.
[4] See *MTAC*, v. 1, p. 6; v. 2, p. 71–2, 210–211.
[5] See *MTAC*, v. 2, p. 211.

367[F].—Mikhail Borisovich Ostrogradskiĭ (1801–1861) *Polnoe Sobranie Sochineniĭ Akademika M. B. Ostrogradskogo* [Complete collected works of Academician M. B. Ostrogradskiĭ], v. 2: *Lektsii Algebraicheskogo i Transtsendentnogo Analiza* [Lectures on algebraic and transcendental analysis], Moscow-Leningrad, Academy of Sciences, 1940, 464 p. 17 × 25 cm. Bound, 19 roubles.

This volume of Ostrogradskiĭ's works contains (p. 433–462) his tables of indices and powers of a primitive root modulo p, for all primes under 200. This set of tables first appeared in Akad. Nauk, S.S.S.R., Leningrad, *Mémoires*, . . . *Sci. Math. Phys. et Nat.* s. 6, v. 3 = *Sci. Math. Phys.*, s. 6, v. 1, "livraison 4," 1836, p. 359–385, and apparently was the first of its kind to be published. These tables were reproduced and extended by Jacobi in 1839 to

$p < 1000$ to form his famous *Canon Arithmeticus* (Compare *MTAC*, v. 1, p. 440). Twelve errata were discovered in Ostrogradskiĭ's tables by Jacobi after the latter had had them set in type:

p	Table	Arg.	*For*	*Read*
71(439)	I	16	15	22
71(439)	I	26	22	15
83(440)	I	25	8	80
127(447)	N	105	107	108
127(447)	N	116	31	71
137(449)	N	108	88	87
167(455)	I	57	128	28
173(456)	I	57	72	92
181(458)	I	16	165	172
181(458)	I	26	172	165
181(458)	N	78	94	64
193(460)	N	155	173	174

More than a century later the tables are now reproduced with the same old errata.

It should be noted that Ostrogradskiĭ's tables give also all the primitive roots of each prime, information not presented in Jacobi's *Canon*.

D. H. L.

EDITORIAL NOTE: Ostrogradskiĭ's portrait is on a plate opposite p. 64 of *Les Mathématiques dans les Publications de l'Académie des Sciences 1728–1935. Répertoire Bibliographique*, Moscow, Academy of Sciences, 1936. In this v., p. 108, the date of publication of Ostrogradskiĭ's "Tables des racines . . ." is given incorrectly as 1838. The blue cover of "livraison 4" in the Harvard University library copy is dated 1836.

368[F].—WILHELM PATZ, *Tafel der regelmässigen Kettenbrüche für die Quadratwurzeln aus den natürlichen Zahlen von 1–10000*. Leipzig, Akademische Verlagsgesellschaft, 1941. Lithoprinted by Edwards Bros., Ann Arbor, Michigan, 1946, xvi, 282 p. 15 × 22.9 cm. $6.50. Published and distributed in the public interest by authority of the Alien Property Custodian under license number A-412.

The regular continued fraction representing the square root of a positive non-square integer has been the subject of much experimental work and theoretical investigation since the time of EULER. This table will serve as a useful tool in the further work along these lines. For each positive non-square $D \leqslant 10002$ are given the periodic partial quotients $b_1, b_2, \cdots, b_p$ in the expansion

$$\sqrt{D} = b_0 + \frac{1}{b_1 +}\,\frac{1}{b_2 +}\cdots+\frac{1}{b_p +}\,\frac{1}{b_1 +}\cdots$$

Here $b_p = 2b_0 = 2[\sqrt{D}]$. Since $b_k = b_{p-k}(k > 0)$, it suffices to give b_k for $k \leqslant \frac{1}{2}p$ and b_p; and this is done except when $p \leqslant 6$. In a majority of cases p is even ($p = 2q$) and b_q is printed with an asterisk. Thus for $D = 178$ and 209 the entries are

$$178|13(2, 1, 12^*, 1, 2, 26), \quad 209|14(2, 5, 3, 2^*, \cdots, 28).$$

In case p is odd a diamond is printed before the expansion. Those values of D which are primes are followed by a small p in the argument column.

The usual method of expanding $\sqrt{D}$ is explained on p. xi–xiii. Writing the nth complete quotient in the form

$$x_n = (\sqrt{D} + P_n)/Q_n = b_n + x_{n+1}^{-1}$$

we have the four formulae of recurrence

$$b_n = (b_0 + P_n)Q_n^{-1},\ P_{n+1} = b_nQ_n - P_n,\ Q_{n+1} = b_n(P_{n+1} - P_n) + Q_{n-1},$$
$$Q_{n+1} = (D - P_{n+1}^2)/Q_n$$

The last of these was used as an "automatic check" on the exactness of the calculation.

It is perhaps worth noting that the first two formulae may be replaced to advantage by the following pair whenever the value of b_n is not at once obvious

$$b_0 + P_n = b_n Q_n + r_n, \quad P_{n+1} = b_0 - r_n,$$

where $0 \leqslant r \leqslant Q$.

The usefulness (and also the number of pages) of this volume would have been more than doubled had the author included the denominators Q_n, as given in the tables of DEGEN,[1] CAYLEY[2] and WHITFORD.[3] These numbers are important in the application of continued fractions, especially to the diophantine equation of Lagrange

$$x^2 - Dy^2 = N$$

Besides this application the importance of this table lies in the wealth of statistical information it gives about the expansion of square roots of integers. The table throws some light on the unsolved questions of whether the period p is even or odd, whether, for a given D, the central partial quotient has the value b_0 or $b_0 - 1$ or not, whether $p < c\sqrt{D}$; and so on. Inspection of the latter part of the table reveals quite a large number of very long expansions. There are 31 values of D for which the period exceeds $2\sqrt{D}$. These range from 1726 to 9949 with periods of 88 and 217 respectively. The ratio $p/\sqrt{D}$ reaches a maximum of 2.2245 at $D = 7606$. Thus there is still room for the conjecture that, for all D, $p < \sqrt{5D}$. These long expansions appear to have more than their share of unit values among their b's. In fact more than 43.62 percent of their partial quotients are equal to unity. The average for all real numbers is only $\log_2 (4/3) = .41503$.

There are three errata listed on p. xiv: $D = 2872$, *for* 1, 2, 2, 4, *read* 1, 2, 4; $D = 4170$, *for* 2, 1, 3, 3, *read* 2, 1, 4, 3; $D = 4966$, *for* 1, 4, 1, 2, *read* 1, 4, 2, 2. The first still occurs in the 1946 edition while the last two have been corrected. Nevertheless the above list is given. This confusing bit of editing led the reviewer to recalculate the expansions for $D = 4170$ and 4966. BEEGER has pointed out (*MTAC*, v. 2, p. 88) that in the 1941 edition the diamond sign is printed one line too low at $D = 6938$, 6949, 6953, and 9698. The first three of these misprints occur also in the present edition but the diamond is two lines too low at 9697 and is missing at 9698. The author has compared his table with those of DEGEN ($D \leqslant 1000$), CAYLEY ($1001 \leqslant D \leqslant 1500$), WHITFORD ($1501 \leqslant D \leqslant 2012$) and THIELMANN[4] (about 140 isolated D's under 10^4). No errata in these tables are quoted although the first and third are known to contain 2 and 4 erroneous continued fraction expansions respectively.[5] The table of ROBERTS[6] for all primes $D = 4n + 1 < 10^4$ was not available to the author. A comparison of these two tables would give a very good idea of the reliability of the one under review.

D. H. L.

[1] C. F. DEGEN, *Canon Pellianus* . . ., Copenhagen, 1817.

[2] A. CAYLEY, "Report of a committee appointed for the purpose of carrying on the tables connected with the Pellian equation from the point where the work was left by Degen in 1817," BAAS, *Report, 1893*, p. 73–120; also *Collected Mathematical Papers*, v. 13, 1897, p. 430–467. [These tables were computed by C. E. BICKMORE.]

[3] E. E. WHITFORD, *The Pell Equation*, New York, 1912, p. 164–190.

[4] M. VON THIELMANN, "Zur Pellschen Gleichung," *Math. Annalen*, v. 95, 1926, p. 635–640.

[5] D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, 1941, p. 138, 171.

[6] C. A. ROBERTS, "Table of the square roots of the prime numbers of the form $4m + 1$ less than 10000 expanded as periodic continued fractions," *Math. Magazine*, v. 2, p. 105–120, 1892.

369[F].—HEINRICH TIETZE, "Einige Tabellen zur Verteilung der Primzahlen auf Untergruppen der Gruppe der teilerfremden Restklassen nach gegebenem Modul," Akad. d. Wiss., Munich, *Abh.*, *Math. Nat. Abt.*, n.s., no. 55, 1944. 31 p. 22.3 × 28.5 cm.

This paper contains 26 short tables giving information about the distribution of primes

in certain sets of arithmetical progressions the last term of which is denoted by L. The actual forms considered are $km + r_j (j = 0, 1, \cdots)$ for the 26 following values of m and r_j:

Table	m	$\phi(m)$	r_j	L
1	10	4	1, 9	571
2	10	4	1	571
3	8	4	1	449
4	8	4	1, 3	457
5	8	4	1, 5	449
6	8	4	1, 7	449
7	9	6	1, 4, 7	487
8	9	6	1, 8	487
9	30	8	1, 29	3511
10	30	8	1, 11	3511
11	30	8	1, 17, 19, 23	1913
12	30	8	1, 7, 13, 19	1879
13	30	8	1, 11, 19, 29	1901
14	26	12	1, 3, 9, 17, 23, 25	1091
15	26	12	1, 3, 9	1069
16	26	12	1, 25	1091
17	26	12	1	1093
18	262	130	Quadratic residues	3931
19	262	130	5th power residues	3929
20	262	130	10th power residues	3911
21	262	130	13th power residues	3929
22	262	130	26th power residues	3467
23	262	130	65th power residues	3929
24	262	130	1	298943
25	262	130	259	298153
26	262	130	17	297911

Under multiplication modulo m, the set of r's in each case forms a group Γ, in fact a subgroup of the group H of the $\varphi(m)$ numbers $\leqslant m$ and prime to m.

Let $\Pi_H(x)$ and $\Pi_\Gamma(x)$ denote the number of primes $\leqslant x$ belonging respectively to H and Γ modulo m. If there are h elements of Γ and if $\phi(m) = hi$, then, according to the prime number theorem (generalized), $\Pi_H(x)$ and $i\Pi_\Gamma(x)$ are asymptotically equal and approach $\phi(m)x/m \ln x$. The tables give values of these step functions together with the difference

$$\Delta(x) = \Pi_H(x) - i\Pi_\Gamma(x)$$

for $x \leqslant L$.

Since $\Delta(x)$ is a step function it suffices to tabulate it only at the values of x where it changes value, that is at primes belonging to Γ (mod m). These primes are denoted by N and form the arguments of the tables. This makes $i\Pi_\Gamma(N)$ merely a list of consecutive multiples of i, and this column might well have been omitted. As x varies from one value of N to the next, $\Delta(x)$ increases because $\Pi_H(x)$ increases. The value of $\Delta(x)$ just before the next value of N is denoted by $\Delta^*(N)$ and is tabulated also.

The modulus 262 is chosen because 131 is the least prime having 3, 5, 7, 11, 13 as quadratic residues, 17 and 259 are the least and greatest primitive roots of 131. The last three large tables, especially table 24 might some day prove useful as a list of primes of these forms. Cunningham's observation that the form $km + 1$ contains fewer primes $\leqslant x$ than $km + l, l \neq 1$, (l, m coprime) does not seem to hold for $m = 262$. In fact $\Delta(N)$ in table 24 changes sign very often.

D. H. L.

370[F].—I. M. VINOGRADOV, *Osnovy Teorii Chisel* [Fundamentals of the Theory of Numbers], Moscow-Leningrad, (**a**) third ed., 10 000 copies, 1940, 111 p. + an errata sheet. 12.7 × 18.7 cm. Bound, 3 roubles. (**b**) Fourth ed., 3 000 copies, 1944, 142 p. + an errata sheet. 13.8 × 20 cm. Paper bound, 4 roubles.

(a) This interesting little volume contains two kinds of tables:

(1) Tables of indices and powers of a primitive root modulo p for $p < 100$ (p. 104–109). These are based on least primitive roots and so are identical with tables of WERTHEIM,[1] and USPENSKY & HEASLET.[2] A comparison with the latter table reveals no discrepancy.

(2) Table of least primitive roots of primes $p < 3000$ (p. 110–111).

Three errata may be noted: $p = 1013$, *for* 2, *read* 3; $p = 2593$, *for* 10, *read* 7; $p = 2999$, *for* 7, read 17.

(b) In this edition the first group of tables (p. 135–140) is the same as in the third edition but the table of least primitive roots of primes < 3000, in the third edition, has been corrected and enlarged to primes < 4000 (p. 141–142).

D. H. L.

[1] G. WERTHEIM, *Aufgangsgründe der Zahlenlehre*. Brunswick, 1902, p. 412–417.
[2] J. V. USPENSKY & M. A. HEASLET, *Elementary Number Theory*. New York and London, 1939, p. 477–480.

371[G, L].—A. COLOMBANI, "La théorie des filtres électriques et les polynomes de Tchebichef," *Jn. de Physique et de Radium*, s. 8, v. 7, Aug. 1946, p. 231–243. 21.3 × 26.6 cm. Compare *MTAC*, v. 1, p. 125, 149f, 385, RMT **381, 383**.

There are two tables for so-called Chebyshev polynomials, p. 236–237. **T. I** gives $S_n(x)$ for $x = -2(.1)0$, $n = [2(1)10; n\mathrm{D}]$. Also zeros of S_n to 5D. **T. II** gives values of $X_n(x) = S_n(x) - S_{n-1}(x)$, for $x_1/x_2 = x - 2 = -4(.1)0$, $x = -2(.1)2$, $n = [1(1)10; n\mathrm{D}]$. Also zeros of $X_{10}(x)$ to 5D. Figs. 2–4, p. 234–235, are graphs of $X_n(x)$ for $n = 1(1)10$.

$$S_n(x) = \sin(n+1)\theta/\sin\theta, \quad x = 2\cos\theta.$$

372[I].—H. E. SALZER, "Coefficients for facilitating the use of the Gaussian quadrature formula," *Jn. Math. Physics*, v. 25, 1946, p. 244–246. 17.5 × 25.5 cm.

In the Gaussian quadrature formula

$$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n} a_i f(x_i) + R_n$$

the sum extends over the roots x_i of the Legendre polynomial $P_n(x)$. As these roots are not equally spaced, it is not possible to test the smoothness of a set of computed ordinates $f(x_i)$ by straight-forward differencing, as one would do in the case of a Cotes type formula.

To examine the $(n-1)$st difference of this set of ordinates one must resort to divided differences. Much of the cumbersome calculation attending the general divided difference process can, in this case, be avoided. In fact this difference can be written

$$\sum_{i=1}^{n} C_i^{(n)} f(x_i)$$

where the coefficients are simply

$$C_i^{(n)} = 2^{-n}\binom{2n}{n}/P_n'(x_i).$$

This paper contains a small table of these coefficients for $n = 3(1)10$. The accuracy is 8D for $n = 3, 4, 5$; 7D for $n = 6, 7, 8$; and 6D for $n = 9, 10$. The author fails to indicate that in his notation the roots x_i are so ordered that

$$x_1 < x_2 < \cdots < x_n,$$

a fact which the user of the table will need to know.

D. H. L.

373[I, L].—H. A. Rademacher & I. J. Schoenberg, "An iteration method for calculation with Laurent series," *Quart. Appl. Math.*, v. 4, July 1946, p. 142–159. 17.5 × 25.4 cm.

In the authors' words, "the purpose of this paper is to describe a method whereby rational or algebraic operations with Laurent series may be performed with high accuracy at the expense of a reasonable amount of labor."

The main problem considered is that of solving numerically

$$f(w, z) \equiv a_0(z)w^m + a_1(z)w^{m-1} + \cdots + a_m(z) = 0 \tag{1}$$

for the coefficients of the Laurent series of a particular branch of $w(z)$, where the $a_i(z)$ are regular and uniform functions of z in the ring

$$R:\quad r_1 < |z| < r_2$$

and where neither $a_0(z)$ nor the discriminant $D(z)$ of (1) is zero in R.

The procedure suggested by the authors is first to find an initial approximation and then to use an iteration scheme based on a modification of Newton's algorithm. In this modification only one division is needed and that is a preliminary one. A method is given of obtaining by trigonometric interpolation a Laurent polynomial,

$$F_n(z) = \sum_{j=-n}^{n} c_{n,j}z^j,$$

as a first approximation. It is proven that $F_n(z)$ approaches the solution as $n \to \infty$. One of the points of this paper is that it is preferable to start with a small value of n and then to iterate rather than to use $F_n(z)$ for a large value of n.

The iteration scheme is as follows: Since the discriminant $D(z)$ is not zero there are polynomials $\phi(w)$ and $\psi(w)$, with coefficients which are polynomials in the $a_i(z)$ divided by $D(z)$, such that,

$$\phi(w)f(w, z) + \psi(w)\frac{df(w, z)}{dw} = 1.$$

The modified Newton's algorithm can now be expressed by the recurrence formula

$$w_{r+1} = w_r - f(w_r, z)\psi(w_r).$$

This has the usual quadratic convergence of the Newton algorithm. The authors state that this method has been used previously by Schwertfeger for the numerical solution of ordinary algebraic and transcendental equations.

The authors show that in the special case of solving

$$a(z)w(z) - 1 = 0$$

the recurrence formula reduces to

$$w_{r+1} = w_r(2 - aw_r),$$

which is the formula described by Hotelling for inverting matrices. Therefore, in reciprocation of a Laurent series one can use an inequality of Hotelling and Lonseth to obtain a limit for the error due to stopping after any number of steps.

As an illustration the authors compute the coefficients w_n of the Laurent expansion of the reciprocal of $-J_0(\sqrt{13z})$ between the first two positive roots of that function. The entire computation to 9D is exhibited in tabular form for w_n with $-29 \leqslant n \leqslant 32$. The remaining coefficients are numerically smaller than 10^{-9}. The paper also contains a description of how the methods of calculating with Laurent series apply to calculations with absolutely convergent Fourier series.

In conclusion one can say that this article presents in a very convenient form a solution to the problem considered, especially for those who will have to do actual computations of this sort.

Abraham Hillman

NBSMTP

374[K].—La Mont C. Cole, "A simple test of the hypothesis that alternative events are equally probable," *Ecology*, v. 26, 1945, p. 204. 16.5 × 25.4 cm.

Table III, values of $P = 2^{1-n}\sum_0^k \frac{n!}{E!(n-E)!}$, for $k = 0(1)12$, $n = [2(1)35; 5D]$. "Table gives proportion in *both* tails of $(\frac{1}{2} + \frac{1}{2})^n$. For larger values of n use $t = (n - 2E)n^{-\frac{1}{2}}$. In general, a value is statistically significant ($P < 0.05$) if $E \leqslant \frac{1}{2}n - n^{\frac{1}{2}}$."

Extracts from text

375[K].—Frederick E. Croxton & Dudley J. Cowden, "Tables to facilitate computation of sampling limits of s, and fiducial limits of sigma," *Industrial Quality Control*, v. 3, July 1946, p. 18–21. 21.6 × 27.9 cm.

For samples of size N drawn from a normal distribution with known variance σ^2, upper and lower percentage points of the distribution of s/σ are given in **Table 1** entitled: "Values of s/σ at selected probability points for various sample sizes." The sample standard deviation is $s = [\sum (x - \bar{x})^2/N]^{\frac{1}{2}}$. The probability points of the distribution of s/σ are given in pairs for probabilities α and $1 - \alpha$, with $\alpha = .001, .005, .01, .025, .05, .10$, for $N = [2(1)30; 3D]$. An approximation is given for $N > 30$. A table similar to the present one for $\alpha = .001, .005$ and for $N = 2(1)15$ is given in Amer. Standards Assoc., *Control Chart Method of Controlling Quality During Production*, no. ASA Z 1.3—1942, p. 40.

Table 2, entitled "Values of σ/s for use in computation of selected fiducial limits of σ for various sample sizes," may be used to obtain confidence or fiducial limits for σ. Confidence levels available are .998, .99, .98, .95, .90, .80 for $N = 2(1)30$. The entries in this table are principally reciprocals of the entries of **Table 1**. A similar table for confidence levels .90, and .98, $N = 5(1)30$, appears in E. S. Pearson, *The Application of Statistical Methods to Industrial Standardisation and Quality Control*, London, British Standards Institution, 1935, p. 69.

The tabulated values of the tables under review were derived principally from Catherine M. Thompson, "Table of percentage points of the χ^2 distribution," *Biometrika*, v. 32, p. 187f, 1941. (See *MTAC*, v. 1, p. 78). But the .999 points of **Table 1** were derived from R. A. Fisher & F. Yates, *Statistical Tables for Biological, Agricultural, and Medical Research*. London, 1938, Table IV, p. 27, while the .001 points of **Table 1** were derived from tables of F shown in F. E. Croxton & D. J. Cowden, *Applied General Statistics*. New York, 1939, p. 878–879.

Frederick Mosteller

Harvard University

Editorial Note: It may be remarked that it was exactly on the pages quoted as sources, Fisher & Yates, 1938, p. 27, and Croxton & Cowden, 1939, p. 878, that we have listed errors in the tables in question, namely: *MTAC*, v. 1, p. 324, and 86.

376[K].—V. L. Goncharov, *Teoriīa Veroīatnosteĭ* [Theory of Probabilities]. Moscow and Leningrad, 1939, 427 p. + errata slip. 14.4 × 21.7 cm. Bound, 11 roubles. An edition of 5000 copies.

This government ordnance industry publication has three small tables on its last seven pages. In the notation of the FMR, *Index*, these are

(a) $H(x) = 2\pi^{-\frac{1}{2}}\int_0^x e^{-t^2}dt$, for $x = [0(.05)2.2; 4D], [2.2(.05)2.75, 3; 6D], [3.5, 4; 9D]$.

(b) $H(\rho x)$, where $\rho = .4769362762\cdots$ is the root of $H(x) = \frac{1}{2}$, for $x = [0(.01)3.4(.1)5.4; 5D]$.

(c) $xH(\rho x) + \rho^{-1}\pi^{-\frac{1}{2}}e^{-\rho^2x^2}$ for $x = [0(.05)5.2; 4D]$.

All three tables give first differences.

Strange to say, the very well known function $H(x)$ is not tabulated correctly in (a). There are two errata: $t = 2.4$, *for* .999312, *read* .999311; $t = 2.7$, *for* .999868, *read* .999866. A last-figure error occurs in (b); in the final entry $t = 5.40$, *for* 99972, *read* 99973.

The function tabulated in (c) is essentially the second iterated integral:

$$\frac{1}{\rho\sqrt{\pi}} + \frac{2}{\sqrt{\pi}}\int_0^x \int_0^{\rho t} e^{-\theta^2} d\theta dt.$$

This table appears to have several last-figure errors. For example, $t = 1.05$, *for* 1466, *read* 1468; $t = 1.10$, *for* 1493, *read* 1494.

For a discussion of the tables the reader may consult p. 150 and 217.

D. H. L.

377[K].—*Mrs.* CATHERINE M. (THOMPSON) GRYLLS & *Mrs.* MAXINE MERRINGTON, "Tables for testing the homogeneity of a set of estimated variances," *Biometrika*, v. 33, June 1946, p. 302–304. A preface by H. O. HARTLEY & E. S. PEARSON occupies p. 296–301. 19.3 × 27.3 cm.

These tables are designed to provide 1 per cent. and 5 per cent. points for testing the hypothesis that the variances of several normal populations, as estimated from two or more independent observations on each, are equal; or hypotheses equivalent thereto. The basis for computation was an approximation of HARTLEY[1] to the distribution of a statistic suggested by BARTLETT[2] which differs in weighting factors from the likelihood ratio statistic proposed by NEYMAN & PEARSON,[3] and which has been found more powerful in certain cases by BISHOP & NAIR.[4]

Hartley's approximation depends on three parameters: the number of populations k, and two functions of the degrees of freedom ν_t of the estimates of the population variances,

$$c_1 = \sum_t \left(\frac{1}{\nu_t}\right) - \frac{1}{N}, \quad c_3 = \sum_t \frac{1}{\nu_t^3} - \frac{1}{N^3}, \quad \text{where} \quad N = \sum_t \nu_t$$

The percentage points vary but little with c_3. The tables are double entry, giving for each pair k and c_1, two values which are approximately the extremes with respect to variation in c_3. An auxiliary table aids interpolation with c_3 in the occasional case when these extremes cover the computed statistic.

The percentage points are given to 2D(3 or 4S) for $k = 3(1)15$ and (the entire range of) $c_1 = 0(.5)5(1)10(2)14$. A historical note, several illustrative examples and a discussion of the accuracy of the approximation, are provided in the preface. It is found that the approximation is "very good" if the degrees of freedom all exceed 2, and is "adequate" if some are as small as 2.

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[1] H. O. HARTLEY, "Testing the homogeneity of a set of variances," *Biometrika*, v. 31, 1940, p. 249–255.

[2] M. S. BARTLETT, "Properties of sufficiency and statistical tests," R. Soc. London, *Proc.*, v. 160A, 1937, p. 268–282.

[3] J. NEYMAN & E. S. PEARSON, "On the problem of k samples," Akad. umiejetności, *Bull. Intern.*, 1931A, p. 460–481.

[4] D. T. BISHOP & U. S. NAIR, "A note on certain methods of testing for the homogeneity of a set of estimated variance," R. Statist. Soc., *Jn.*, v. 102, *Suppl.* v. 6, 1939, p. 89–99.

378[K, V].—CECIL HASTINGS & MARGARET PIEDEM, *Miscellaneous Probability Tables*, calculated and checked under the direction of Dr. H. H. GERMOND, 1942–1944. Applied Mathematics Panel, National Defense Research Committee, Note no. 14, New York, July 1944. ii, 65 p. Offset print. 21.3 × 27.8 cm. These tables are not available for public distribution.

The main table (p. 6–37 and introduction p. 1–5) is devoted to the two-dimensional normal distribution function (or error function)

$$V(h, q) = (2\pi)^{-1} \int_0^h \int_0^{qx/h} e^{-\frac{1}{2}W} dy dx, \; W = x^2 + y^2.$$

The double entry table to 5D has h and q/h as independent variables, the former appearing in rows, the latter in columns; $h = 0(.01)4$, $q/h = .1(.1)1$. Differences are tabulated in the main rows and columns between the corresponding entries. The differences within columns have usually only one digit and never exceed 85 units; the differences in the rows keep to three digits. The need for the tables arose in probability problems associated with bombing and fragmentation damage. NBSMTP supplied the key values of $V(h, q)$ from which the Table was subtabulated. The computation was carried on for some time before the appearance of C. NICHOLSON, "The probability integral for two variables," *Biometrika*, v. 33, part 1, April, 1943, p. 59–72, where a table of $V(h, q)$, to 6D, is given for $h = .1(.1)3$, $q = .1(.1)3, \infty$.

Pages 39–45 cover tables for $H(x) - xH'(x)$, where $H(x)$ is defined by

$$H(x) = 2(\pi)^{-\frac{1}{2}} \int_0^x e^{-t^2} dt.$$

The range is [0(.001)3(.01)4.3; 7D]; for $x > 4.3$ the entries would equal unity throughout. (The entries in the first column on p. 45 are misprinted: the last zero should be deleted everywhere, and the last line on p. 44 should be deleted. This table was computed by using NBSMTP, *Tables of Probability Functions*, v. 1, 1941.) It is stated that the rounding errors will occasionally amount to one unit in the last digit. There follow, p. 46–49, tables of the inverse of the function $H(x) - xH'(x)$ covering the entire significant range, namely [0(.001)1; 5D].

The next table, p. 51–62, gives values of the function $y = 1 - (1 + x)e^{-x}$. These tables will be useful in particular in connection with the Poisson distribution. The range is [0(.001)5(.01)10(.1)15; 5D]. Again, y is practically constant for $x > 15$.

Two small tables conclude the collection. On p. 63 we find values of the product xy where y is defined by the equation

$$xy = 1 - e^{-y}.$$

The range is x = [0(.01)1; 5D], Δ^2. Finally, on p. 65 is a table of

$$\phi(x) = xe^{-x^2} \int_0^x e^{t^2} dt$$

for the range [2(.1)7; 5D], Δ^2.

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EDITORIAL NOTE: In our notes on Dawson's or Poisson's integral we have listed an earlier table of $\phi(x)$, *MTAC*, v. 1, p. 323, N. KAPZOV & S. GWOSDOWER, *Z. f. Physik*, v. 45, 1927, p. 133. This table is for the range x = [.1, .5, .8(.2)1.2(.05)2.2; 5D]; Δ, 1.5–2.2. The use for such a table there, arose in discussion of oscillations in electron tubes.

379[L].—D. CHALONGE & V. KOURGANOFF, "Recherches sur le spectre continu du soleil," *Annales d'Astrophysique*, v. 9, 1946, p. 69–96. 21.5 × 27.4 cm.

Appendix I contains two tables of "la fonction Γ incomplète d'argument négatif":

$$\tilde{\Gamma}_x(\alpha) = \int_0^x t^{\alpha-1} e^{+t} dt$$

T. A, p. 94. x = [0(.01).1; 4D]; α = [.1(.1)1],

T. B, p. 95–96. α = [.01(.01)1; 4D]; x = [0(.1)1.1].

The authors state that the tables constitute "un extrait, pour le domaine qui nous intéresse ici, d'une table plus étendue qui paraîtra prochainement." No details of the calculation are given.

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380[L].—HARVARD UNIVERSITY, Computation Laboratory, *Annals*, v. 3: *Tables of the Bessel Functions of the First Kind of Orders Zero and One;* v. 4: *Tables of the Bessel Functions of the First Kind of Orders Two and Three*, by the Staff of the Laboratory, Professor H. H. AIKEN, Technical Director, Cambridge, Mass., Harvard Univ. Press, 1947. ii, xxvii, 652 p. and viii, 652 p. 19.5 × 26.7 cm. $10.00 + $10.00. Compare *MTAC*, v. 2, p. 176f, 185f. The offset printing of these volumes is of outstanding excellence.

P. iii, "Staff of the Computation Laboratory" [11 members and 13 assistants listed].

P. vi, "Preface" by Professor Aiken. Quotations: In Nov. 1944 a conference at the Naval Research Laboratory was called to discuss the tabulation of Bessel functions of the first kind and of high order, which at that time were needed in connection with various research problems of interest to the Navy. As a result the computation project of the Bureau of Ships was directed to tabulate the required functions. During discussion it became clear that if $J_{100}(x)$ was to be accurate to ten decimal places, an adding, multiplying, and storage capacity of more than forty digits would be required of the Automatic Sequence Controlled Calculator. Since the multiplying unit of the calculator already supplied forty-six digits and the algebraic sign, it was only necessary to link two normal storage registers, each comprising twenty-three digits and the algebraic sign, to form a single adding storage register covering forty-six digits and the algebraic sign.

On the first of January 1946, the Computation Project was transferred to the Bureau of Ordnance.

P. ix–xxii, "Introduction" by RICHARD M. BLOCH

Part I. The Bessel Functions, p. ix–xi,
Part II. The Computation of the Tables, p. xii–xviii,
Part III. Interpolation in the Tables, p. xix–xxii.

Part I. Because of the important applications of Bessel Functions to many physical phenomena, they have been the subject of intensive investigation for many years. Recent advances in the theory of frequency modulations, resonance in cavities, waves in various media, vibration theory of structures, and other problems of physics and engineering have greatly increased the need for extensive tables of $J_n(x)$ and $Y_n(x)$ covering a large range both of the order and of the argument.

The Staff of the Computation Laboratory is at present engaged in the tabulation of $J_n(x)$, $0 \leqslant x < 100$, $n = 0(1)100$. The present volumes contain tables for $n = 0(1)3$, $x = [0(.001)25(.01)99.99; 18D]$.

The Bessel functions satisfy the two relations

$$J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$$

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x). \tag{1}$$

For machine computation, the successive application of the recurrence formula (1) provides the most feasible method of obtaining the high order functions. Since ten decimals place accuracy is to be maintained in the tables of $J_n(x)$ for $4 \leqslant n \leqslant 100$, it was necessary to investigate the cumulative loss of accuracy which arises in the repetitive use of (1) as the

computation proceeds. If $J_{m-1}(x)$ and $J_m(x)$ ($m = 3$ for $x < 2$, $m = 2$ for $x \geqslant 2$) are the two basic functions upon which the recurrence is constructed, the maximum number of decimal places lost is eleven. Consequently the low order functions $J_0(x)$, $J_1(x)$ and $J_2(x)$ were computed correct to twenty-three places of decimals. Since the figures were available, the tables of $J_n(x)$ ($n = 0, 1, 2, 3$) have been printed to 18D, despite the fact that interpolation within these tables to full accuracy would be extremely difficult with the present manual aids to computation.

The Automatic Sequence Controlled Calculator is so arranged that after all final results are automatically checked, they are printed by the typewriters controlled by the machine itself. Certain tables of Bessel functions to eighteen or more D were read against the values computed at the Computation Laboratory. These comparisons were made with the values of the functions listed in the following three tables: MEISSEL, as in GRAY, MATHEWS, & MACROBERT, *A Treatise on Bessel Functions*, 1931, p. 286–299, 18D, $1 \leqslant x \leqslant 24$, $\Delta x = 1$, $n = 0(1)3$; HAYASHI, *Tafeln der Besselschen, Theta-, Kugel-, und anderer Funktionen*, 1930, p. 52–59, $n = 0(1)3$, $.01 \leqslant x \leqslant 100$, selected Δx, 22–103D; ALDIS, R. Soc. London, *Proc.*, v. 66, 1900, p. 40–43, $n = 0, 1$; $0 \leqslant x \leqslant 6$, $\Delta x = .1$, 21D. No discrepancies were observed. ***Part II.*** Values of $J_n(x)$ given in other tables were not used. All numerical constants including the coefficients of the ascending power series, the asymptotic series and those required for interpolation, were evaluated at the Computation Laboratory, regardless of the availability of such material from external sources.

The ascending power series

$$J_n(x) = \sum_{r=0}^{\infty} k^{2r+n} a_{r,n} \left(\frac{x}{2k}\right)^{2r+n},$$

where $a_{r,n} = (-1)^r/r!(n+r)!$, and k is a normalizing factor, was used to evaluate $J_n(x)$ for $n = 0(1)3$ over the range $0 \leqslant x < 2$ with increment $\Delta x = .001$, and for $n = 0(1)2$ over the range $2 \leqslant x \leqslant 25$ with increment .01. The values of $10^m \cdot a_{r,n}$ computed to 50D, are given for $r = 1(1)60$, $0 \leqslant m \leqslant 168$, $n = 0(1)2$.

For the range $0 \leqslant x < 2$, $k = 1$; $2 \leqslant x < 10$, $k = 5$; $10 \leqslant x < 20$, $k = 10$; $20 \leqslant x \leqslant 25$, $k = 20$. For $n = 3$ we have $|10^m \cdot a_{r,3}|$, $r = 0(1)15$, $1 \leqslant m \leqslant 28$, p. xxiii–xxxi.

There are similar tables (p. xxxii–xxxvii) for the various asymptotic expansions.

Extracts from introductory text

381[L].—C. W. JONES, J. C. P. MILLER, J. F. C. CONN, & R. C. PANKHURST, "Tables of Chebyshev polynomials," R. Soc. Edinb., *Proc.*, v. 62A, no. 21, 1946, p. 187–203. 17.5 × 25.5 cm.

The main object of the article under review is to present a table of the Chebyshev polynomials $C_n(x) = 2\cos(n \arccos \frac{1}{2}x)$ for $n = 1(1)12$ and $x = 0(.02)2$. The tabulated values are either exact or given to 10D. In addition to the table of $C_n(x)$, the article contains also short tables of the functions $(4 - x^2)^{\frac{1}{2}}$, $(2 + x)^{\frac{1}{2}}$, $(2 - x)^{\frac{1}{2}}$ and $\arccos \frac{1}{2}x$ required in the applications of Chebyshev polynomials discussed in Dr. Miller's article, "Two numerical applications of Chebyshev polynomials" (RMT **383**).

The tabular material is preceded by an excellent introduction giving the definition of the Chebyshev polynomials $C_n(x)$ and $S_n(x)$ and of other related functions, the differential equations and recurrence relations satisfied by these functions, the explicit power series expressions of these functions, the expressions of the twelve powers of x in terms of Chebyshev polynomials $C_n(x)$, the orthogonality relations and the generating functions for each of the functions under consideration.

The reviewer agrees with the authors' remarks that the tables will be of particular importance to computers. One application particularly worth mentioning is the process of interpolation by means of Chebyshev polynomials (RMT **383**). The efficacy of this process of interpolation is illustrated by the following observation: In a certain region of the Mathieu functions ms. in preparation by the NBSMTP, interpolation to the full accuracy of the table would require the use of a formula involving differences up to the ninth order; the

corresponding interpolation formula in terms of $C_n(x)$ requires only the first five Chebyshev polynomials.

Forty percent of the entries of the table under review were proofread against the corresponding entries in the more extensive table of Chebyshev polynomials prepared by the NBSMTP (see *MTAC*, v. 1, p. 125); no discrepancies were discovered.

ARNOLD N. LOWAN

382[L].—N. W. McLACHLAN (**a**) "Computation of the solution of Mathieu's equation," *Phil. Mag.*, s. 7, v. 36, June 1945 (publ. Jan. 1946), p. 403–414. 17 × 25.5 cm. (**b**) "Mathieu functions and their classification," *Jn. Math. Phys.*, v. 25, Oct., 1946, p. 209–240. 17.3 × 25.4 cm.

(**a**) This paper deals with the computation of the solutions of

$$\frac{d^2y}{dz^2} + (a - 2q\cos 2z)y = 0 \tag{1}$$

where a, q are real parameters. For certain characteristic values, the solutions of (1) are periodic, of period π or 2π. Those characteristic values which give rise to even solutions of period π and 2π are denoted by a_{2m} and a_{2m+1}, respectively. The characteristic values giving rise to odd solutions of period 2π and π are denoted by b_{2m+1} and b_{2m+2}, respectively. The curves $a = a_m(q)$ and $a = b_m(q)$ separate the $(a - q)$ plane[1] into regions in which the solutions are "stable" or "unstable." When the parametric point (a, q) lies between a_m and b_{m+1}, the solution is "stable"; that is, two independent solutions of (1) may be written in the form

$$y = \sum_{r=-\infty}^{\infty} c_r \frac{\cos}{\sin} (2r + p + \beta)z. \tag{2}$$

In (2), $p = 0$ if the subscript m in a_m is even, and $p = 1$ if m is odd. According to currently accepted theory, there exists a unique positive value of β less than unity corresponding to every point (a, q) in this region, such that the solution (2) remains finite as z approaches infinity through real values.

The value of a which, for a fixed q, determines β, will be denoted by $a_{m+\beta}$. Between $a_m(q)$ and $b_{m+1}(q)$ there lies a family of iso-β curves, i.e., β = constant. When β turns out to be a proper fraction p/s (in its lowest terms), the solution will be periodic, of period $2\pi s$. When β is irrational, the solutions of (2) will be non-periodic.

If the parametric point (a, q) lies between b_m and a_m, the solutions are "unstable"; that is, no solution of the form (2) exists. By Floquet's theorem, there does exist a solution of the form

$$y = e^{\mu z} \sum_{r=-\infty}^{\infty} c_r e^{(2r+p)zi}, \tag{3}$$

where $p = 0$ if m is even in b_m, a_m, and $p = 1$ if m is odd. When (a, q) lies in an unstable region, μ is real. It may be readily seen that when μ is a purely imaginary number, (3) yields the solutions (2).

The most important contribution of the paper is to show how β may be determined, in the stable region, from a knowledge of the characteristic values a_m and b_m. Let a point (a, q) of a stable region be given and let it be desired to determine β. The author improves upon Ince's method by obtaining some good *first approximation* to β. By inverting the known series for "a" in terms of $(m + \beta)$ and q, the author obtains the following approximation[1] to β:

$$\beta = \left[a - \frac{(a-1)q^2}{2(a-1)^2 - q^2} - \frac{(5a+7)q^4}{32(a-1)^3(a-4)} \cdots \right]^{\frac{1}{2}} - m. \tag{4}$$

In (**b**) the above expansion is extended to include, in the radical, the term $-(9a^2 + 58a$

$+ 29)q^6/64(a-1)^5(a-4)(a-9)$. This reviewer believes that, if the first three terms are left in their present form, the term involving q^6 should be[2] $(-37a^3 + 319a^2 - 587a + 17)q^6/64(a-1)^5(a-4)^2(a-9)$. However, the series (4) is useful only when the expression under the radical converges rapidly enough with the given terms; and whenever the contribution from the corrected term involving q^6 is small, the uncorrected term will not be much larger numerically.

When (4) cannot be used (a denominator may vanish or $q/(a-1)$ may be too large), then other approximations may be used. Let a_m and b_{m+1} be the characteristic values for the given q, between which a lies. Let $\lambda = (a - a_m)/(b_{m+1} - a_m)$. Then λ should be an approximation to β. However, usually λ is too crude; a better approximation may be obtained if it is assumed that, at $q = 0$, the iso-β curve intersects the region between a_m and b_{m+1} in approximately the same ratio, λ. Since, at $q = 0$, $a_m = m^2$, and $b_{m+1} = (m+1)^2$, it follows from (4) and the above assumption as to λ that

$$(5) \qquad \beta = m\left\{\left[1 + \lambda\left(\frac{2m+1}{m^2}\right)\right]^{\frac{1}{2}} - 1\right\}.$$

The author gives still another empirical formula for β. Let

$$\varphi_m = (a^{\frac{1}{2}} - a_m^{\frac{1}{2}})/(b_{m+1}^{\frac{1}{2}} - a_m^{\frac{1}{2}}).$$

Replacing λ by φ_m in (5), one obtains

$$(6) \qquad \beta = m\left\{\left[1 + \varphi_m\left(\frac{2m+1}{m^2}\right)\right]^{\frac{1}{2}} - 1\right\}.$$

In an appendix, the author gives a table showing the accuracy of the several approximations in a few of the instances in which they were tried. The schedule given below summarizes the author's results; all figures except those in the last column were recalculated by this reviewer; in cases of discrepancy, the recalculated figures are given. The results obtained were close to those of the author, except on line 2 of the schedule, where the author obtained, by formula (4), an amount .57—apparently by neglecting the third term under the radical.

				Formulae Used			
m	a	q	λ	(4)	(5)	(6)	More Accurate Value of β
1	3	2	.48	—	.56	.59	.579
2	8	4	.48	.52	.53	.55	.59
2	6	2	.21	[3].34	.25	.27	.34
5	36	16	.48	[3].62	.50	.51	[3].583
1	2	.0125	.33	.4142	.41	.49	.4142
0	−1.45	2	.52	—	.72	.72	.52

The last two examples were supplied by the reviewer. Except in the last example, formulae (4), (5), and (6) are better than λ, and (4) gives a good approximation in many cases. The author recommends the use of this formula wherever possible. It is to be noted that the author's ingenious method of improving on λ in (5) and (6) is fruitful in its results, especially when (4) cannot be used.

Once an approximation to β has been obtained, the method of improving it by iteration or interpolation, in the process of computing the coefficients c_r, is fairly easy. Thus in the recurrence relation

$$(7) \qquad [a - (2r + p + \beta)^2]c_r - q(c_{r+1} + c_{r-1}) = 0,$$

one may neglect $c_{\pm(r+1)}$ for r sufficiently large, and compute in turn $c_{r-1}, \cdots, c_0$ in terms of c_r (hence also $c_r, \cdots, c_1$ in terms of c_0); and again $c_{-r+1}, \cdots, c_0$ in terms of c_{-r} (hence also c_{-r+1}, etc. in terms of c_0). Then setting $r = 0$ in (7), the relation between c_{-1}, c_1, and c_0 will be satisfied only if β is correct, and the divergence of the right-hand side of (7) from zero shows how to correct β and the coefficients c_r. One may of course compute the coefficients in terms of any c_m rather than in terms of c_0. Several variations of the computing technique are given, with methods of checking the computations.

The method of approximation may also be used for the unstable regions, once iso-μ

curves have been plotted. It is recommended that the coefficients c_r be normalized so that $\sum c_r^2 = 1$.

(b) Here are given a great variety of representations for the solutions of Mathieu's equation and of Mathieu's "modified" equation. Quoting from the author: "The number of representations in the guise of series, integral relations, etc., exceeds 300. Of these, about 200 have not been published hitherto. . . . No attempt is made to show the derivation of the new formulae, as this paper would then be much too long."

The first part of the paper deals with solutions of the first and second kind for Mathieu functions of integral order, and includes the very useful Bessel-function-products solutions, previously given in a paper by W. G. BICKLEY & N. W. MCLACHLAN, *MTAC*, v. 2, Jan. 1946, p. 1–11. It may be worth pointing out that Bessel-function-products solutions of Mathieu's differential equation were given by BRUNO SIEGER[4]; and although Sieger's work is again mentioned by STRUTT,[5] the importance of such solutions seems to have been little understood until it was emphasized by Bickley & McLachlan in their paper. (This reviewer learned of Sieger's work from Professor Bickley.) All forms given in the January paper are included in this larger one by McLachlan, now under review. In addition, a great many variations of the Bessel-function-products are given, which may prove useful from a computational standpoint.

Functions of the third kind (analogous to the well-known Hankel functions) are defined in section 11. Except for the normalization factor, these solutions are the same as the ones previously defined by L. J. CHU and J. A. STRATTON (*Jn. Math. Phys.*, Aug., 1941), see *MTAC*, v. 1, p. 157. The relations between the various solutions, when the parameter q is either positive or negative, are also given.

In addition to formulae relating to Mathieu functions of fractional order (stable solutions) the author devotes considerable space to the "unstable" solutions. It is shown that, when the solution is put into the form

$$y_1 = e^{\mu z} \sum_{r=-\infty}^{\infty} c_{2r+p} e^{(2r+p)zi},$$

(with $p = 0$ if (a, q) lies between a_{2m} and b_{2m} and $p = 1$ if (a, q) lies between b_{2m+1} and a_{2m+1}) then c_{2r} and c_{-2r} are conjugate complex numbers, if expressed in terms of c_0, real; furthermore if $p = 1$, then c_{2r+1} and $(c_1/c_{-1})c_{-2r-1}$ are conjugate, if c_1 is taken to be real. It is shown that there exists a real solution of the differential equation which tends to zero as $z \to -\infty$ through real values, if a, q, and $\mu(> 0)$ are real. Solutions denoted by $ce_{m+\mu}(\pm z, q)$ are defined, analogous to the solutions $ce_m(\pm z, q)$ for integral m; similarly for $se_{m+\mu}(\pm z, q)$. Such solutions are neither even nor odd. It is shown how to construct even and odd solutions, but they are less useful than $ce_{m+\mu}(\pm z, q)$ and $se_{m+\mu}(\pm z, q)$.

For the same q and μ, there are *two* values of a. One is such that the solution approaches $ce_m(z, q)$ as $\mu \to 0$; the second a corresponds to the solution which approaches $se_m(z, q)$ as $\mu \to 0$. Solutions of fractional order corresponding to Mathieu's modified equation are also given.

The paper contains a number of asymptotic expansions for Mathieu functions of integral order, both for large z and large q. These expansions involve certain multipliers (resulting from the normalization adopted) which cannot readily be expressed asymptotically. To this extent the solutions for large q, in a practical case, are really never obtainable by the given asymptotic formulae—they are known *except* for those multipliers which are functions of q.

It is this unfortunate property of the normalization adopted by McLachlan (and the English school generally) which is at the crux of the divergence of opinion, regarding the normalization scheme, between the English and American schools.

The author concludes with a section on the zeros of the functions and another on the classification of the various solutions of integral order, fractional order, and the unstable solutions. A useful iso-μ chart, the data for which are credited to Dr. L. J. COMRIE, is also given.

The paper is concisely written and represents a prodigious effort, both as to span

covered and the variety of formulae given. It forms a priceless compendium of known results (with a very considerable portion of them due to the author himself). The summary given above by no means covers all topics treated. It is hoped that the book promised by the author, enlarging on the theory covered in this paper, may soon be forthcoming.

GERTRUDE BLANCH

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[1] In the author's diagrams, the horizontal lines in the a–q plane are parallel to the "q" axis. Hence it might have been better to refer to the "q–a" plane, and to the parametric point as (q, a). We shall not, however, depart from the author's notation.

[2] Mrs. IDA RHODES of the NBSMTP checked this reviewer's inversion. Dr. McLachlan pointed out that the second term of (4), given incorrectly in (**a**), is correct in (**b**).

[3] These three entries are corrections of the printed values, furnished by Dr. McLachlan. EDITOR.

[4] B. SIEGER, "Die Beugung einer ebenen elektrischen Welle an einem Schirm von elliptischem Querschnitt," *Ann. d. Phys.*, s. 4, v. 27, 1908, p. 626–664.

[5] M. J. O. STRUTT, *Lamésche-Mathieusche und verwandte Funktionen der Physik u. Technik*, (*Ergebnisse d. Math.*, v. 1, no. 3), Berlin, 1932, p. 46–48.

383[L].—J. C. P. MILLER, "Two numerical applications of Chebyshev polynomials," R.S. Edinb., *Proc.*, v. 62, no. 22, 1946, p. 204–210. 17.5 × 25.7 cm.

1. The strong convergence of an expansion in Chebyshev polynomials renders them useful for interpolation. Let $f(a + t)$ be expanded into a series of $C_n(4t/h)$. The coefficients a_n of this expansion are expressible in terms of the derivatives $f^{(p)}(a)$. They are also expressible in terms of central differences. The corresponding formulae are derived by operational methods and numerical tables given. Even if high central differences are needed, the smaller number of a_m terms makes interpolation more convenient, particularly in conjunction with a table for the $C_n(x)$. A numerical example illustrates the advantage of the method.

2. Since an expansion in Chebyshev polynomials is merely a modified form of a Fourier series, a table of the Chebyshev polynomials becomes useful for harmonic synthesis whenever the sum of a Fourier series is required for arguments which are convenient numbers in $x = 2\cos\theta$ rather than in θ itself. An application is given, showing how the evaluation of the Mathieu functions can be facilitated by this procedure.

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384[L].—FRANK H. SLAYMAKER, WILLARD F. MEEKER & LYNN L. MERRILL, "The directional characteristics of a free-edge disk mounted in a flat baffle or in a parabolic horn," Acoustical Soc. Amer., *Jn.*, v. 18, Oct. 1946, p. 363–368. 19.4 × 26.6 cm.

There are two tables on p. 367. **T. I** of

$$\phi(\lambda_n r) = \frac{I_1(\lambda_n a)}{J_1(\lambda_n a)} J_0(\lambda_n r) + I_0(\lambda_n r)$$

$r/a = 0(.1)1$; $\lambda_n a \sim 3.01, 6.21, 9.37$ for $n = 1(1)3$; $n = 1, 2$ to 3S, and $n = 3$ mostly to 4S.

$$\begin{aligned} G &= J_1(ka\sin\phi)/ka\sin\phi; \\ P_n &= -I_1(\lambda_n a)[\lambda_n a J_1(\lambda_n a)J_0(ka\sin\phi) - ka\sin\phi J_0(\lambda_n a)J_1(ka\sin\phi)] \\ &\quad \div [J_1(\lambda_n a)(\lambda_n^2 a^2 - k^2a^2\sin^2\phi)], \text{ if } \lambda_n \neq k\sin\phi \\ &= -I_1(\lambda_n a)[J_0^2(\lambda_n a) + J_1^2(\lambda_n a)]/[2J_1(\lambda_n a)], \text{ if } \lambda_n = k\sin\phi; \\ Q_n &= [\lambda_n a I_1(\lambda_n a)J_0(ka\sin\phi) + ka\sin\phi I_0(\lambda_n a)J_1(ka\sin\phi)] \\ &\quad \div (\lambda_n^2 a^2 + k^2a^2\sin^2\phi). \end{aligned}$$

T. II is of G, $P_n + Q_n[n = 1(1)3]$, mostly to 3S, for $ka\sin\phi = 0(.5)10(1)12$. There are graphs of $P_1 + Q_1$, $P_2 + Q_2$, $P_3 + Q_3$ on p. 368.

R. C. A.

385[L].—FRANCESCO TRICOMI, "Generalizzazione di una formula asintotica sui polinomi di Laguerre e sue applicazioni," Accad. delle Scienze di Torino, *Cl. d. sci. fis., mat., e nat., Atti*, v. 76, 1941, p. 288–316. 16.6 × 25 cm.

The Tricomi polynomials of Laguerre[1] are defined by

$$\begin{aligned} L_n(t) &= (n!)^{-1}e^t d^n(e^{-t}t^n)/dt^n = M(-n, 1, t) \\ &= (-1)^n(n!)^{-1}\left[t^n - \frac{n^2}{1!}t^{n-1} + \frac{n^2(n-1)^2}{2!}t^{n-2} - \cdots + (-1)^n n!\right]. \\ &= 1 - \binom{n}{1}\frac{t}{1!} + \binom{n}{2}\frac{t^2}{2!} - \cdots + (-1)^n\frac{t^n}{n!} \end{aligned}$$

This is the case where $a = 0$ in the more general formula

$$L^a_{n+a}(t) = (n!)^{-1}e^t t^{-a} d^n(e^{-t}t^{n+a})/dt^n = M(-n, a+1, t).$$

In Tricomi's paper are the following:
p. 292, values of $e^{-\frac{1}{2}t}L_{10}(t)$, for t = [.5(.5)3(1)8(2)34; 5D], Δ;
p. 302, a 4D table of the roots of the equations $x + \sin x = a$, for a = 0(.1)3(.02)3.18;
p. 303, graph of zeros of $L_n(t)$ for n = 1(1)10;
p. 315–316, table of $e^{-\frac{1}{2}t}L_n(t)$, n = 1(1)10, t = [.1(.1)1(.25)3(.5)6(1)14(2)34; 4D].

The polynomials $L_n(t)$ and their properties were given in E. N. LAGUERRE, "Sur l'intégral $\int_x^\infty e^{-x}dx/x$," *Bull. Sci. Math.*, v. 7, 1879, p. 72–81; *Oeuvres de Laguerre*, Paris, v. 1, 1898, p. 428f ($L_n(t)$, p. 430). $L^a_{n+a}(t)$ seems to have been discussed simultaneously with $L_n(t)$, by N. SONIN, in a memoir dated Aug. 1879, *Math. Annalen*, v. 16, 1880, p. 41 (function T_m^n). The first one to refer to $L^a_{n+a}(t)$ as generalized Laguerre polynomials appears to have been another Russian, WERA MYLLER-LEBEDEFF, *Math. Annalen*, v. 64, 1927, p. 410.

The so-called polynomials of Laguerre were introduced into mathematical analysis by LAGRANGE,[2] more than 130 years earlier than Laguerre, in his solution of a dynamical problem in which the oscillations of a vertical chain are represented approximately by those of a set of similar weights equally spaced on a light string. (See H. BATEMAN, "Lagrange's compound pendulum," *Amer. Math. Mo.*, v. 38, 1931, p. 1–8.) The polynomials were also considered by ABEL, in 1826, "Sur une espèce particulière de fonctions entières nées du développement de la fonction $(1-v)^{-1}e^{-xv/(1-v)}$ suivant les puissances de v."[3] This function is equal to $\sum_k L_k(x)v^k/k!$. In H. BETHE, "Quantenmechanik der Ein- und Zwei-Elektronenprobleme," *Handbuch der Physik*, second ed., v. 24_1, Berlin, 1933, p. 289, this result is attributed to E. SCHRÖDINGER,[4] just 100 years later.

Laguerre polynomials are also of use in (a) the theory of hydrogen-like atoms; (b) the problem of numerical integration over the range 0 to $+\infty$ [Gauss's method with Legendre polynomials for ranges -1 to $+1$, or 0 to 1; Hermite's polynomials for the range $-\infty$ to $+\infty$]; (c) the discussion of the mathematical foundations of the electromagnetic theory of the paraboloidal reflector.[5] In Bateman's Bibliography[5] there are 47 references for $L_n(x)$, and 73 for "$L_n{}^a(x)$."

In preparing this RMT I have been indebted for some assistance from Dr. J. C. P. MILLER, and from Dr. ALAN FLETCHER.

R. C. A.

[1] See *MTAC*, v. 1, p. 361, 425; and v. 2, p. 31 [where $L_n(x)$ is defined without the factor $(n!)^{-1}$], 89.

[2] J. L. LAGRANGE, "Solution de différents problèmes de calcul intégral," *Miscellanea Taurinensia*, v. 3, 1762–65; *Oeuvres*, v. 1, Paris, 1867, "Des oscillations d'un fil fixe par une de ses extrémités, et chargé d'un nombre quelconque de poids," p. 534–536; there are four of the polynomials on p. 536.

[3] N. H. ABEL, *Oeuvres Complètes*, Christiania, 1881, v. 2, p. 284.

[4] E. SCHRÖDINGER, *Annalen d. Physik*, v. 385, 1926, p. 485.

[5] PINNEY, *Jn. Math. Phys.*, v. 25, 1946, p. 49f. Harry Bateman's Bibliography, p. 77–79.

386[L, M].—S. A. KHRISTIANOVICH, S. G. MIKHLIN & B. B. DAVISON, *Nekotorye novye voprosy mekhaniki sploshnoĭ sredy* [Some new questions in mechanics of a continuous medium]. Moscow and Leningrad, Akad. N., Matematicheskiĭ Institut imeni V. A. Steklova, 1938. 407 p. 16.7 × 25.3 cm.

We shall list certain tables in this volume p. 274–336 of a section written by DAVISON, and p. 392–395 of an appendix to the work, presumably written by the joint authors.

On p. 274 a table is given of

$$k\pi x/q = \tan^{-1}\sqrt{e^{2\pi ky/q}-1} - (1/\mu)\sqrt{e^{2\pi ky/q}-1}$$

for $k\pi y/q = [0(.2)3; 5S]$, $1/\mu = 1(1)4$.

$$K(t) = \int_0^{\frac{1}{2}\pi}\frac{d\theta}{\sqrt{1-t\sin^2\theta}}, \qquad \tfrac{1}{2}\gamma(t) = \tan^{-1}\frac{K(1-t)}{K(t)}.$$

On p. 334 there is a table of $\frac{1}{2}\gamma(t)$, to 3D, for $t = 0(.00001).0001(.0001).0005(.0002).0015$, $.002(.001).01(.003).016$, $.02(.005).05(.01).1(.02).2(.1).5$.

On p. 335–336 are the following five tables:

$$\text{(a) } z = \int_0^1\frac{\gamma(t)dt}{t-\lambda}, \qquad \text{(b) } e^{-2z/2\pi}, \qquad \text{to 3–4S,}$$

for $-\lambda = .01, .1, .2, .5(.1).7, 1(1)3, 5, 8, 15, 20$, and $\lambda = 1.05, 1.1, 2(1)8, 12, 15, 20$.

$$\text{(c) } \gamma(\lambda), \qquad \text{(d) } w = R\left[\int_0^1\frac{\gamma(t)dt}{t-\lambda}\right], \qquad \text{(e) } |e^{-2w/2\pi}|,$$

mostly to 3–4S, for $\lambda = .00001, .005, .01, .05(.05).3, .4(.05).6, .7(.05).95, .99, .995, .9999, .99999$. In (a) and (d) approximations are given to true results with possible maximum deviations. Before discussing the remaining tables in the volume we may quote some results from B. A. BAKHMETEV, *Hydraulics of Open Channels*, New York, 1932, p. xv–xvi, 308–311: If y is the depth or stage of flow, y_0 the normal depth of flow or the depth of flow in uniform movement, and $\eta = y/y_0$, then the varied flow function $B(\eta) = -\int_0^\eta\frac{d\eta}{\eta^n-1}$, n being the hydraulic exponent. There are tables of $B(\eta)$ (f) for $\eta > 1$, (g) for $\eta < 1$, $n = 2.8(.2)4.2(.4)5.4$

(f) $\eta = 1.001, 1.005(.005)1.02(.01)1.2(.02)1.5(.05)2(.1)3(.5)5(1)10, 20$
(g) $\eta = 0(.02).6(.01).97(.005).995, .999$.

Now on p. 392–395 are tables of

$$\text{(h) } A(\eta) = \int_{\eta_0}^{\eta}\frac{\eta^{\frac{1}{2}(k-1)}}{\eta^k-1}d\eta \qquad \text{(k) } C(\eta) = \int_{\eta_0}^{\eta}\frac{\eta^{k-\frac{1}{2}}}{\eta^k-1}d\eta,$$

$k = 2n+1$, for $k = 3(n = 1)$, $4(n = 3/2)$, $\eta = [0(.05).6(.01).9(.005)1.05(.01)1.5(.05)2(.1)3(.5)5, 6(2)10; 4D]$, Δ.

By trial we found that in the table of $A(\eta)$, when $k = 3$, $\eta_0 = 0$, for $\eta < 1$, and $\eta_0 = \infty$ for $\eta > 1$. We were unable to determine the values of η_0 leading to the table of $C(\eta)$, or to that of $A(\eta)$, when $k = 4$.

S. A. J. & R. C. A.

387[L, M].—NBSMTP, "Table of the Struve functions $L_\nu(x)$ and $H_\nu(x)$," *Jn. Math. Phys.*, v. 25, Oct. 1946, p. 252–259. 17.3 × 25.3 cm. This is Applied Mathematics Panel, *Report* 59.1, referred to in *MTAC*, v. 2, p. 39.

The functions in question are, in Watson's notation, in effect,

$$L_n(x) = \sum_{m=0}^{\infty} a_m(x); \qquad H_n(x) = \sum_{m=0}^{\infty}(-1)^m a_m(x),$$

where

$$a_m(x) = (\tfrac{1}{2}x)^{2m+1+n}/\Gamma(m + \tfrac{3}{2})\Gamma(m + n + \tfrac{3}{2}).$$

Also

$$L_n(x) = N \int_0^{\frac{1}{2}\pi} \sinh(x \cos \theta)\sin^{2n}\theta d\theta, \qquad N = 2(\tfrac{1}{2}x)^n/\Gamma(n + \tfrac{1}{2})\Gamma(\tfrac{1}{2}),$$

$$H_n(x) = N \int_0^{\frac{1}{2}\pi} \sin(x \cos \theta)\sin^{2n}\theta d\theta, \qquad \text{provided that } R(n) > -\tfrac{1}{2}.$$

But for all n

$$L_n(x) = A_n(x) + B_n(x), \qquad H_n(x) = A_n(x) - B_n(x),$$

where

$$A_n(x) = \sum_{m=0}^{\infty} a_{2m}(x), \qquad B_n(x) = \sum_{m=0}^{\infty} a_{2m+1}(x).$$

Tables are given of $L_n(x)$ and $H_n(x)$, $n = 0, -1, -2$, $x = [0(.1)10; 7\text{–}10S]$. $L_0(x)$, $L_{-1}(x)$, are also with δ^2 and $*\delta^4$ modified throughout the range; $L_{-2}(x)$ with δ^2 and $*\delta^4$ for $x = 2.1(.1)10$ For $x = 0(.1)2$, $L_{-2}(x)$, $xL_{-2}(x)$, $\delta^2(xL_{-2})$, $*\delta^4(xL_{-2})$ are given. In the case of $H_0(x)$ and $H_{-1}(x)$, and $H_{-2}(x)$, for $x = 2(.1)10$, $*\delta^2$ is given; $H_{-2}(x)$ for $x = 0(.1)2$, also $xH_{-2}(x)$, has $*\delta^2$.

$$H_1(x) = 2/\pi - H_{-1}(x), \qquad L_1(x) = -2/\pi + L_{-1}(x).$$

WATSON[1] has tabulated $H_0(x)$ and $H_1(x)$ $x = [0(.02)16; 7D]$, no Δ; hence linear interpolation is here correct to about 5D. $H_{-1}(x)$ is readily obtained from $H_1(x)$, and has been tabulated before by AIREY,[2] for $x = [0(.02)16; 6D]$, and by JAHNKE & EMDE,[3] for $x = [0(.01)14.99; 4D]$. $H_{-2}(x)$, $L_{-1}(x)$, and $L_{-2}(x)$ seem to be here independently tabulated for the first time.

KARL HERMANN STRUVE investigated[4] only the special functions $H_0(x)$ and $H_1(x)$, but properties of the general function were later extensively developed by SIEMON[5] and WALKER.[6] The function $L_n(x)$ bears the same relation to Struve's function $H_n(x)$, as $I_n(x)$ bears to $J_n(x)$; $L_n(x) = i^{-n-1}H_n(ix)$, $L_0(x) = -iH_0(ix)$; $L_0'(x) = 2/\pi - H_1(ix)$. Tables of $L_0(x)$ and $L_0'(x)$ for $x = [.02, .1, .5, 1(1)12; 6\text{–}7S]$ were given by OWEN.[7] $H_n(x) = (-i)^{n+1}L_n(ix)$, and

$$H_n(xi^{\frac{3}{2}}) = i^{n+1}L_n(xi^{\frac{1}{2}}) = \text{ster}_n x + i\,\text{stei}_n x,$$

a notation due to McLACHLAN & MEYERS[8] (see *MTAC*, v. 1, p. 252, 460). For tables of $y = \frac{1}{2}\pi[I_0(x) - \mathbf{L}_0(x)]$, and $-y' = 1 - \frac{1}{2}\pi[I_1(x) - \mathbf{L}_1(x)]$ by R. ZURMÜHL and R. MÜLLER, see *MTAC*, v. 2, p. 59; and on p. 39 a table by GREAT BRITAIN, Nautical Almanac Office, of $f(x) = \pi e^{-x}(2x)^{-1}[I_1(x) + L_{-1}(x) - 2/\pi]$.

When n is half an odd positive integer $H_n(x)$ is expressible in terms of elementary functions. For example, $H_{\frac{1}{2}}(x) = B(1 - \cos x) = B - J_{-\frac{1}{2}}(x)$, where $B = [2/(\pi x)]^{\frac{1}{2}}$. For various tables of $J_{\frac{1}{2}}(x) = H_{-\frac{1}{2}}(x)$, see *MTAC*, v. 1, p. 233.

R. C. A.

[1] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, p. 328–329; tables, p. 666–697.

[2] J. R. AIREY, BAAS, *Report, 1924*, p. 280f, $-H_{-1}(x)$ is also tabulated here for the same range.

[3] JAHNKE & EMDE, *Tables of Functions*, fourth ed., New York, 1945, p. 219f. $H_0(x)$ is also tabulated here for the same range, p. 212f, and 218f. There are also two other tables of $H_0(x)$ and $H_1(x)$, to 4D, by S. P. GLAZENAP, *Matematicheskie i Astronomicheskie Tablīsy*, Leningrad, 1932, p. 110f, $x = 0(.02)16$, an abridgment of Watson; and by N. W. McLACHLAN, *Bessel Functions for Engineers*, 1934, p. 176, $x = 0(.1)15.9$.

[4] See *MTAC*, v. 1, p. 305.

[5] P. SIEMON, *Ueber die Integrale einer nicht homogenen Differentialgleichung zweiter Ordnung*. Progr. Luisenschule. Berlin, 1890; see *Jahrb. Fort. d. Math.*, 1890, p. 340f.

[6] J. WALKER, *The Analytical Theory of Light*, Cambridge, 1904, p. 392f.

[7] S. P. OWEN, "Table of values of the integral $\int_0^x K_0(t)dt$," *Phil. Mag.*, s. 6, v. 47, 1924, p. 736; see also *MTAC*, v. 1, p. 245, 247, 301.

[8] See also N. W. McLACHLAN & A. L. MEYERS, (a) "The ster and stei functions" (b) "Integrals involving Bessel and Struve functions," *Phil. Mag.*, s. 7, v. 21, 1936, p. 425–436, 437–448.

388[L, M].—S. SKOLEM, "En del bestemte integraler av formen $\int_0^\infty f(x)\cos akdx$ og $\int_0^\infty f(x)\sin(ax)dx$," *Norsk Matem. Tids.*, v. 27, 1945, p. 65–75; tables, p. 70–71. 15.5 × 23.2 cm.

$$S(a,1) = \int_0^\infty \frac{\sin ax}{1+x^2}\,dx = \tfrac{1}{2}[e^{-a}Ei(a) - e^{a}Ei(-a)]$$
$$= \cosh a \operatorname{shi} a - \sinh a \operatorname{chi} a$$
$$= 1/a + 2!/a^3 + 4!/a^5 + \cdots + (2n)!/a^{2n+1}$$
$$S'(a,1) = -\tfrac{1}{2}[e^{-a}Ei(a) + e^{a}Ei(-a)] = \sinh a \operatorname{shi} a - \cosh a \operatorname{chi} a$$
$$= -[1/a^2 + 3!/a^4 + 5!/a^6 + \cdots].$$

T. I, $S(a, 1)$, for $a = [0(.01).1(.1)1(1)10(10)100; 5D]$; maximum value at $a \sim .8791$ is approximately .64996.

T. II, $S'(a, 1)$, for the same range of a as in **T. I**; zero value at $a \sim .8791$, and minimum value at $a \sim 1.8594$ is approximately $-.15583$.

389[L, M].—EDMUND C. STONER, "The demagnetizing factors for ellipsoids," *Phil. Mag.*, s. 7, v. 36, Dec. 1945 (publ. Sept. 1946; note added in proof 28 May 1946), p. 803–821. 17 × 25.4 cm.

J. A. OSBORN, "Demagnetizing factors of the general ellipsoid," *Phys. Rev.*, v. 67, 1945, p. 351–357. 19.2 × 26 cm.

The formulae for the demagnetizing factors, in terms of F and E, are

$$D_a = L/4\pi = \frac{abc}{(a^2-c^2)^{\frac{1}{2}}(a^2-b^2)}[F(k,\phi) - E(k,\phi)]$$
$$= \frac{\cos\theta\cos\phi}{\sin^3\phi\sin^2\alpha}[F(k,\phi) - E(k,\phi)]$$

$$D_b = M/4\pi = -\frac{abc}{(a^2-c^2)^{\frac{1}{2}}(a^2-b^2)}[F(k,\phi) - E(k,\phi)]$$
$$+ \frac{abc}{(a^2-c^2)^{\frac{1}{2}}(b^2-c^2)}E(k,\phi) - \frac{c^2}{b^2-c^2}$$
$$= \frac{\cos\theta\cos\phi}{\sin^3\phi\sin^2\alpha\cos^2\alpha}\left[E(k,\phi) - \cos^2\alpha F(k,\phi) - \frac{\sin^2\alpha\sin\phi\cos\phi}{\cos\theta}\right]$$

$$D_c = N/4\pi = -\frac{abc}{(a^2-c^2)^{\frac{1}{2}}(b^2-c^2)}E(k,\phi)$$
$$+ \frac{b^2}{b^2-c^2} = \frac{\cos\theta\cos\phi}{\sin^3\phi\cos^2\alpha}\left[\frac{\sin\phi\cos\theta}{\cos\phi} - E(k,\phi)\right],$$

where $k^2 = (a^2 - b^2)/(a^2 - c^2) = \sin^2\alpha$, $\cos\theta = b/a$, $\cos\phi = c/a$; a, b, $c(a \geqslant b \geqslant c)$ are the semi-axes of the ellipsoid. $D_a + D_b + D_c = 1$.

Consider first, ellipsoids of revolution: a polar semi-axis, b equatorial semi-axis, $m = a/b$, $\mu = b/a$; $m < 1$ and $\mu > 1$ an oblate spheroid; $m > 1$ and $\mu < 1$ a prolate spheroid. Then

$$D_a = \tfrac{1}{2}ab^2\int_0^\infty \frac{ds}{(a^2+s)^{\frac{3}{2}}(b^2+s)} = \frac{1}{(m^2-1)}\left[\frac{m}{(m^2-1)^{\frac{1}{2}}}\cosh^{-1}m - 1\right], \quad m > 1,$$
$$= \frac{1}{(1-m^2)}\left[1 - \frac{m}{(1-m^2)^{\frac{1}{2}}}\cos^{-1}m\right], \quad m < 1;$$
$$D_b = \tfrac{1}{2}(1 - D_a).$$

Stoner gives tables (p. 816–817) of D_a, m or $\mu = [0(.1)5(.5)10(1)25(5)50(10)150(50)400(100)1300; 6D]$. In general the m-table will be appropriate for prolate spheroids and the μ-table for oblate spheroids. To ensure accuracy to the sixth place, the calculations were carried out so as to give unit accuracy in the seventh place, and rounded six-place values are presented in the tables.

Osborn gives two tables (p. 353–354) of demagnetizing factors of the general ellipsoid, $L/4\pi$, $M/4\pi$, $N/4\pi$, for (T. I) $\cos\phi(= c/a)$, $\phi = 10°(10°)70°(5°)85°, 88°, 89°$, and for $\cos\theta\ (= b/a)$, $\theta = [0(10°)90°; 5D]$. Also (T. II) $\cos\theta = .1(.1)1$, $\cos\phi$ = various values. There are three large-scale graphs of $L/4\pi$, $M/4\pi$, $N/4\pi$, for $0 \leqslant c/a \leqslant 1$, $b/a = 0(.1)1$. In T. II the values are accurate to 3D and are probably in error several units in the fourth place.

Stoner has a single graph of these same functions for $b/a = .2(.2)1$.

R. C. A.

390[L, P].—N. W. McLachlan, *Bessel Functions for Engineers*, Oxford Univ. Press, London, Geoffrey Cumberleye, 1946. xii, 192 p. Reprinted photographically. 15.3 × 23.3 cm. 18 shillings.

This very useful volume of the Oxford Engineering Science Series first appeared in 1934. In the corrected photographic reprint published in London in 1941, two pages were added; the new material included an introductory "Note," a page of "Additional formulae," and 20 (instead of 6) "Additional references." In the Note it is remarked that "The omission of contour integral representation of Bessel functions and its technical applications has been rectified through publication [by the author] in 1939 of *Complex Variable and Operational Calculus with Technical Applications*," and a correction of an error on p. 300 of this volume is noted.

We have already referred to various tables in the volume under review (see *MTAC*, v. 1, p. 212, 216, 220, 246, 247, 254, 255, 257, 258, 297). In the right-hand member of formula 147, p. 167 one error still persists; the sign − should be changed to +. The 1946 edition is an exact reprint of that of 1941, except for the correction of four signs, two in each of the lines − 3 and − 5, p. xi, "Additional Formulae."

R. C. A.

391[L, S].—C. Strachey & P. J. Wallis, "Hahn's functions $S_m(\alpha)$ and $U_m(\alpha)$," *Phil. Mag.*, s. 7, v. 37, Feb. 1946 [publ. Nov. 1946], p. 87–94. 16.8 × 15.1 cm.

"In a paper[1] on the calculation of fields in certain resonators, Hahn introduced two new functions:

$$-S_m(\alpha) = \sum_{n=1}^{\infty} \frac{m^2 \sin^2 n\pi\alpha}{n(m^2 - n^2\alpha^2)}, \quad \text{and}$$

$$U_m(\alpha) = \sum_{n=1}^{\infty} \frac{\alpha^2 m^2 n^2 \sin^2 n\pi\alpha}{(m^2 - n^2\alpha^2)^2}, \quad \text{with} \quad 0 < \alpha < 1,$$

and used these functions to shorten his calculations. Since this time, Hahn's method has been used for certain similarly-shaped resonators and Hahn's two functions usually help to shorten the solution considerably. Hahn himself only gave a small table of $S_m(\alpha)$ and a few values of $U_m(\alpha)$.[1]

"In this report closed expressions are derived for the case of α rational, and are used to produce a much more comprehensive table of $S_m(\alpha)$ and a slightly smaller table of $U_m(\alpha)/m$. In a concluding section integral expressions, power series in α, and asymptotic series in m are given which together facilitate the calculation for values of α not given in the tables."

Tables: $-S_m(\alpha)$, for $m = 1(1)10$, $\alpha = [0(.1)1, .25, .75, \frac{1}{3}, \frac{2}{3}; 5D]$;
$U_m(\alpha)/m$, for $m = 1(1)10$, $\alpha = [0(.25)1, \frac{1}{3}, \frac{2}{3}; 5D]$.

Extracts from text

[1] W. C. Hahn, "A new method for the calculation of cavity resonators," *Jn. Appl. Phys.*, v. 12, 1941, p. 62–68. There are 2D values of $-S_m(\alpha)$ for $m = 1(1)9$, $\alpha = 0(\frac{1}{8})\frac{3}{8}(\frac{1}{16})\frac{5}{8}(\frac{1}{8}), 1$; also $-S_0(\alpha)$ for $\alpha = 0(\frac{1}{8})\frac{1}{2}, \frac{7}{16}$; and of $U_m(\frac{1}{2})$, $m = 1(1)4$. See *MTAC*, RMT 208 and MTE 69, v. 1, p. 425, 451.—Editors.

392[M].—NATIONAL RESEARCH COUNCIL OF CANADA, Division of Atomic Energy. Report no. MT-1 dated Chalk River, Ontario, December 2, 1946, *The Functions* $E_n(x) = \int_1^\infty e^{-xu}u^{-n}du$, 39 leaves mimeographed on one side, with covers. Introductory material, p. 1–7 by G. PLACZEK; Appendix A, an asymptolic expansion for $E_n(x)$, by Dr. GERTRUDE BLANCH, p. 8; Tables, by NBSMTP, p. 9–39. 20.3 × 27.4 cm. This edition contains corrections of one which appeared in July–August 1946.

The functions $E_n(x)$ play an important role in diffusion theory. The discussion of certain integral equations can be simplified by their use; expansions in terms of these functions are also often found convenient for the numerical evaluation of integrals occurring in connection with transport problems. The functions have been defined by SCHLÖMILCH,[1] and have been extensively used by SCHWARZSCHILD,[2] EDDINGTON,[3] HOPF,[4] and others. In spite of this no systematic effort for their tabulation seems to have been made up to the present. An attempt by MIAN & CHAPMAN[5] to approximate the functions by "index sums" was not accurate enough for our purposes.

$E_n(x)$ is here tabulated, for $n = 0(1)20$, $x =$ [0(.01)2; 7D], [2(.1)10; 7–10D]. On p. 39 are tables of $E_2(x) - x \ln x$, for $x =$ [0(.01).5; 7D], and of $E_3(x) + \frac{1}{2}x^2 \ln x$ for $x =$ [0(.01).1; 7D], for use in interpolation. Since $E_1(x) = -Ei(-x) = \int_x^\infty e^{-u}u^{-1}du$, there are extensive tables of this function in NBSMTP, *Tables of Sine, Cosine, and Exponential Integrals*, v. 1–2, 1940, for $x =$ [0(.0001)2; 9D], [0(.001)10; 9S]; [10(.1)15; 14D].

Extracts from introductory text

EDITORIAL NOTES: In FMR, *Index*. p. 207, are given details of 8 tables, $E_{-n}(x) = \int_1^\infty e^{-xu}u^n du = x^{-(n+1)}\int_x^\infty e^{-u}u^n du$, five of them including negative values of n. $E_0(x) = e^{-x}/x$, of which values for $x =$ [.1(.001)1(.01)2; 9D] are given by W. L. MILLER & T. R. ROSEBRUGH, "Numerical values of certain functions involving e^{-x}," R. Soc. Canada, *Trans.*, s. 2, v. 9, 1903, sect. III, p. 102–107. See also TAKEO AKAHIRA, "Tables of e^{-x}/x and $\int_x^\infty e^{-u}du/u$, from $x = 20$ to $x = 50$," Inst. Phys. Chem. Research, Tokyo, *Sci. Papers*, Table no. 3, 1929, p. 180–215; the interval of the table is .02, to 5–6S, Δ^2.

[1] O. SCHLÖMILCH, "Ueber Facultätreihen," *Z. Math. u. Phys.*, v. 4, 1859, p. 390f.
[2] K. SCHWARZSCHILD, (a) "Ueber das Gleichgewicht der Sonnenatmosphäre," Gesell. d. Wissen., Göttingen, *Nach.*, *Math-phys. Kl.*, 1906: p. 41f; (b) "Über Diffusion und Absorption in der Sonnenatmosphäre," Akad. d. Wissen., Berlin, *Sitzb.*, 1914, p. 1183f.
[3] A. S. EDDINGTON, *The Internal Constitution of the Stars*, Cambridge, 1926, p. 333.
[4] E. HOPF, *Mathematical Problems of Radiative Equilibrium* (*Cambridge Tracts* . . ., no. 31), 1934, p. 21, etc.
[5] A. M. MIAN & S. CHAPMAN, "Approximate formulae for functions expressed as definite integrals," *Phil. Mag.*, s. 7, v. 33, 1942, p. 115f. It is noted that $E_n(x)$ arises in the theory of absorption of radiation in an exponential atmosphere. There is a table on p. 119 of approximate values of $E_n(x)$, for $n = 2(1)8$, for $x = 0(.5)3(1)6$, also .01, .05, .1, .25.

393[M].—W. SOKOLOVSKY, "Plastic plane stressed states according to Mises," Akad. N., USSR, Leningrad, (*Dok.*), *C. R.*, n.s. 1946, v. 51, p. 177. 16.8 × 26 cm.

There is here a table of

$$-\Omega(x) = \frac{1}{2}\int_{\frac{1}{2}\pi}^{x} \frac{R(t)dt}{\sin t}$$

$= \frac{1}{4}\pi - \sin^{-1}(2\cos x/3^{\frac{1}{2}}) + \frac{1}{4}\tan^{-1}[(4\cos x + 3)/R(x)] + \frac{1}{4}\tan^{-1}[(4\cos x - 3)/R(x)]$, where $R(x) = (3\text{–}4\cos^2 x)^{\frac{1}{2}}$, for $x = \frac{1}{6}\pi$ to $\frac{5}{6}\pi$, mostly at interval $\frac{1}{18}\pi$, to 3D.

394[M].—A. J. C. WILSON, "The integral breadths of Debye-Scherrer lines produced by divergent X rays," Phys. Soc., London, *Proc.*, v. 58, July 1946, p. 407. 18 × 26 cm.

There is given here a table of $D(u) = 4u^{-4}\int_0^u[C^2(u) + S^2(u)]udu$, for $u =$ [0(.1)5; 4D], 5D for $u < 1$. For $u < 2$ the values were calculated from the series for $D(u)$, those for

$u > 2$ by numerical integration of four-place tables of $C(u)$ and $S(u)$. In the range .5 to 2 the greatest difference between the values calculated by the two methods is .0003; the mean difference is about .0001.

Extracts from text

395[N].—ERICH MICHALUP, "Beitrag zur Amortisationsrechnung," *Skandinavisk Aktuarietidskrift*, 1946, p. 80–84. 15.5 × 23.5 cm.

With references to earlier discussions by E. LINDELÖF, K. A. POUKKA, A. BERGER, R. PALMQVIST, H. HOLME, and E. FRANCKX, the author considers the following five formulae and gives tables for each of them to 7D for half-yearly, quarterly, monthly rates ($p = 2, 4, 12$), for $i = 1\%(1\%)9\%$:

$$a_p \sim \frac{1}{p}\left(1 - \frac{p-1}{2p}\,i\right), \qquad a_p \sim \frac{1}{p}\left(1 - \frac{p-1}{2p}\,i + \frac{(p-1)(2p-1)}{6p^2}\,i^2\right),$$

$$a_p \sim 1\Big/\left(p + \frac{p-1}{2}\,i\right), \qquad a_p = [(1+i)^{1/p} - 1]/i, \qquad a_p \sim \frac{1}{p}\left[\frac{6p + i(1+p)}{6p + 2i(2p-1)}\right].$$

396[Q].—ENRIQUE VIDAL ABASCAL, *El Problema de la Órbita Aparente en las Estrellas Dobles Visuales*: Diss. Spain, Consejo Superior de Investigaciones Cientificas, Instituto Nacional de Geofisica, no. 6, Observatorio de Santiago, *Publicaciones*, II, Santiago de Compostela, 1944. xvi, 62 p. 21.2 × 27 cm.

Consider ellipses with common major semi-axis, $OA = 1$, and eccentricities $e =$ the length of $OF_i = .1(.1).9$; then the foci F_i, $i = 1(1)9$, divide OA into tenths. Suppose that a unit circle, with center at O, has been drawn, and P is any point of the circumference, then F_iP and F_iA are the sides of circular sectors, F_iAPF_i, whose angle α may increase from 0 to 360°. A table, p. 53–62, gives the area of such sectors, to 4D, $e = .1(.1).9$, $\alpha = 0(1°)360°$.

R. C. A.

397[U].—FRANCISCO RADLER DE AQUINO, "*Universal" Nautical and Aeronautical Tables. Uniform and Universal Solutions Ultra-simplified.* Rio de Janeiro, Imprensa Naval, 1943, 18, 247 p. 17.5 × 24.5 cm. Copies of this volume may be had from Weems System of Navigation, Annapolis, Md. at $9.00.

The author of these tables, a captain in the Brazilian Navy, is well known to navigators around the world, having published more than fifty papers on navigation in the past forty-eight years. Not so well known is the fact that he was born in New York City on January 23, 1878; his mother was an American, his father a Brazilian. He moved to Brazil at the age of 13 and entered the Brazilian Naval Academy at 15.

This volume is the second Brazilian edition of a book which was first published in Rio de Janeiro in 1903. Editions were published in London in 1910, 1912 with reprintings in 1917 and 1918, and in 1924; and in Annapolis, Md. in 1927 and 1938. The title and content of the tables have changed slightly from edition to edition. For those familiar with the earlier editions, it may be said that the principal change in this edition is the reduction of the interval of the argument, latitude, from 1° to 10′. The method and the tables continue to be universal in that they allow the determination of the altitude and azimuth whatever the values of latitude, hour angle, declination and altitude.

The first eighteen pages in this volume include the title page and explanation of the tables in English; the next sixteen pages (numbered 1 to 16 also) present similar but not identical material in Portuguese. The principal table was designed to be used in a solution of the astronomical triangle in which a perpendicular is dropped from the zenith upon the

hour circle through the celestial body. The length of the perpendicular and the declination of its foot, are called a and b respectively. The angle at the zenith between the perpendicular and the meridian (toward the elevated pole) is called α; that between the perpendicular and the great circle from the zenith to the celestial body is called β. L, t, and d denote the latitude of the observer, the local hour angle and the declination of the celestial body respectively. A is the angle of the astronomical triangle at the celestial body. C is the angular distance from the celestial body to the foot of the perpendicular.

The basic equations for the solution of the two right triangles are obtained by Napier's rules; they are:

$$\csc a = \sec L \csc t; \qquad \tan b = \tan L \sec t$$
$$\tan \alpha = \csc L \operatorname{ctn} t; \qquad \csc h = \sec a \sec C$$
$$\tan A = \tan a \csc C; \qquad \tan \beta = \tan C \csc a$$

The rules necessary for the use of the equations and the tables are given on the pages numbered 8 in the explanations in English and Portuguese. At first sight, they appear to be of the same order of complexity as those in H.O. 208, Dreisonstok (RMT **103**), but use proves them to be somewhat simpler.

The principal table (p. 36–215) has as vertical argument the local hour angle, t, $0(10)90°$, and as horizontal argument the latitude, L, $0(10')89° 50'$. The tabulated quantities are a and b as defined above, each to the nearest minute of arc, and α. The values of a are given in heavy type to distinguish them from those of b. On the right-hand side of the page, α is given to the nearest tenth of a degree for each degree of local hour angle and for the middle of the degree of latitude.

One enters the table with the dead-reckoning latitude rounded off to the nearest ten minutes of arc and with the local hour angle of the body to the nearest degree; a and b are copied out for these arguments and α is taken for the nearest half degree of latitude. C is formed using the equation, $C = |b - d|$. One re-enters the table looking for a (rounded off to the nearest $10'$) at the *bottom of the page* and C (to the nearest degree) along the *right-hand side of the page*. With these arguments, the dark-faced column yields the altitude, h, and β is found in the right-hand column opposite it. The azimuth angle is found by adding α and β.

In the explanation, the author indicates that the determination of β by this method is weak and proceeds to give three other methods of finding it. The first two involve the substitution of arguments; one can look for $90° - C$ in the β column and interpolate the value of β in the left-hand column above B, or one can interchange the values of a and C and interpolate the value of β in the column footed A. The third method is to use the last equation above with a table of log tangents and log secants. Actually the table given is one of values of $10^5 \log \tan x$ and $10^5 \log \sec x$ to the nearest integer for argument x, $0(1')89° 59'$.

To allow for the minutes discarded in a and C, Aquino suggests the use of:

$$\Delta h_1 = \Delta a \cos \beta, \qquad \Delta h_2 = \Delta C \cos A.$$

He provides a "difference of latitude and departure" table to simplify their use; the table will also be useful in dead reckoning. To allow for the minutes of latitude discarded, another table is provided on the inside of the back cover and the page facing it. This same table is offered on the two sides of a separate sheet of cardboard and again on one side of a separate folded sheet of heavy paper. The corrections, Δh_1 and Δh_2, may be avoided by the use of the equation for $\csc h$, and the log tangent − log secant table.

The author states that the tables were computed by means of Callet's and Bagay's seven-place logarithms, with many values determined by Vlacq's ten-place logarithms; Vega's ten-place table, based on Vlacq, is much more accurate. He further states that if the declination be taken to the nearest tenth of a minute, and Δh_1 and Δh_2 used, the maximum possible error in h will be $1.6'$ but that the actual error in practise will hardly ever be over $0.5'$. If only Δh_2 is used, the altitude obtained is always within $5'$ of the true calculated altitude. Although the volume under review is dated 1943 there is in it a yellow sheet dated 9 April 1946, listing 32 corrigenda.

Other tables contained in the volume are four-place logarithms and antilogarithms with convenient proportional parts tables, distance to the horizon and dip of the horizon for different elevations, combined corrections for refraction, dip of the horizon and, where they are significant, semidiameter and parallax for planets and stars, upper and lower limb of the sun and lower limb of the moon, for altitudes 8° to 90° and for elevations 0 to 15 meters. A similar table for the upper limb of the moon would make a worth-while addition. A small auxiliary table allowing one to correct altitudes less than 8° for refraction is a valuable item, not often found in navigation tables and especially needed in the polar latitudes.

A table that remains in use almost half a century while other tables come and go, can reasonably be said to have a strong appeal to the average navigator. To appreciate Aquino's great contribution to navigation, one needs only to compare the first edition of this table which appeared in 1903 with other tables and methods then in use. It is unlikely that a person who has been trained in the use of H.O. 214 or H.O. 218 will change to this table, but a person who learned Aquino's method first might continue to prefer it because of its beautiful simplicity, its universality and the small bulk of the tables.

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398[U].—W. MYERSCOUGH & W. HAMILTON, *Rapid Navigation Tables*. London, Pitman, 1939. ii, 109 p. 16.4 × 26.6 cm. 10s. 6d.

These tables are designed for the solution of the astronomical triangle for altitude, h, and azimuth, Z, when latitude, L, hour angle, t, and declination, d, are known, and for similar problems. In the procedure for the determination of h, Myerscough & Hamilton follow such tables as those by SOUILLAGOUET, OGURA, WEEMS (RMT **315**), DREISONSTOK (RMT **103**), and HUGHES-COMRIE (RMT **115**), each of which divides the astronomical triangle at the zenith into two right triangles. The first triangle is solved for tabular values of L and t by a table which gives the remaining parts, as angles or as logarithmic functions of angles, without interpolation or other calculation. The second triangle is solved for h by logarithmic processes. The several tables differ only in notation and in that one of the auxiliary angles used in some of the tables is the complement of that used in the others.

In the determination of Z the several tables cited show a pleasing variety in method. Dreisonstok and Comrie follow BERTIN in deriving the two component parts of Z from the same auxiliary triangles as are used in the determination of h. Souillagouet utilizes another division of the astronomical triangle in order to get Z in one piece. Weems uses the graphical "Rust diagram," and in his *New Line of Position Tables* (RMT **315**) provides also an interesting variation on the Bertin procedure. Myerscough and Hamilton, however, follow Ogura in using the equation,

$$\cot Z = \cos L\,(\tan d \csc t - \tan L \cot t)$$

The most interesting and original feature of Myerscough & Hamilton is the inclusion of all data in one table of 91 pages (0 to 90°). At the first entry the page is selected for t, and for the left-hand argument L or d, as the case may be, the following quantities are extracted:

P = length of side of first auxiliary triangle opposite zenith, deg. & min.,
$Q = 10^5 \log \sec$ (side of same triangle opposite pole),
$X = 10 \tan L \cot t, \qquad Y = 10 \tan d \csc t.$

For the second entry P is combined with d according to rules typical of such tables to give a side of the second auxiliary triangle. The page being for the degrees of this argument, and the entry by the minutes (using the same figures as were previously used for L and d), the following quantity is taken from the R-column of the table: $R = 10^5 \log \csc (P \sim d)$. For the third entry the R-column is searched for $Q + R = 10^5 \log \csc h$, and h is obtained by reading degrees at the top of the selected page and minutes in the left-hand column. The equivalence of this procedure to those of the other tables cited is easily recognized. For

the fourth entry into the tables the page is selected for the latitude, and the Z-column is searched for $Y - X = 10 \cot Z \csc L$, and the azimuth is read opposite the nearest value. The four necessary openings equal those required by the other tables cited, so that the prospective user must seek grounds for preference in the arrangement of the tables, which is entitled at least to study by other table makers.

The table would be easier to use if it gave the four values of t for which a given page is used and not merely the one in the first quadrant. It would be improved also if the L and d argument went from 0 to 90° instead of stopping at 70°. (With these changes and two other minor ones the Y-column might be used in the fourth step, in order to eliminate the Z-column.) Since the tables are entered with L in the fourth step, there would be nothing gained by rearranging the tables for entry with L in the first, as there is in Hughes-Comrie and the new Weems. No data on the accuracy of the table are available.

The tables of Myerscough & Hamilton and of the other authors cited above seek to avoid interpolation in the second auxiliary triangle by the use of logarithmic trigonometric functions. While there may be some historical justification for such a treatment, it should be pointed out that H.O. 214 has accustomed navigators to interpolation. There is, accordingly, good reason to reexamine the possibilities of such methods as that of Bertin, in which both triangles are solved by a single table. The four openings of the various tables cited above are reduced to two, with two interpolations of about the same magnitude as those customary with H.O. 214 (RMT **399**). The Bertin method is possible with the well-known *Sea and Air Navigation Tables* of Captain Radler de Aquino (RMT **397**), and with a new *Spheric Tabulations* of R. C. Dove, R. F. D. No. 1, Collegeville, Penna.

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399[U].—U. S. Hydrographic Office, *Publication No. 214, Tables of Computed Altitude and Azimuth, Latitudes 80° to 89°, inclusive, Vol. 9.* Washington, D. C., U. S. Government Printing Office, 1946, 3, xxiv, 263 p. 22.6 × 29.1 cm. This is the last of nine uniform v. of H.O. 214, each v. devoted to 10° of latitude. For sale by the Hydrographic Office and by the Superintendent of Documents, Washington, D. C., $2.25 per v.; foreign price, postage extra.

This review will be limited primarily to a discussion of the differences between v. 9 and the other 8 v. of H.O. 214 which were reviewed earlier (RMT **105**). This volume, like the others, was prepared by the Work Projects Administration, (Philadelphia Project No. 24831), and presumably is of the same order of accuracy (see v. 2, p. 182f). The interval of argument for hour angle is 1° as in the other v.; it might well be 2° or perhaps even 5°, save for the loss of uniformity, since the tabulated altitude and azimuth change slowly and in a relatively linear fashion.

The description of the tables and their use is almost entirely new and occupies some ten pages more than that in the other v. The use of the pole as an assumed position is explained as well as the use of gnomonic, stereographic, azimuthal equidistant and inverse Mercator projections. A brief description of grid navigation is given.

Two ways in which this volume could be improved may be mentioned. The computed altitudes might be carried down to 0° or at least to 2°, since in the polar regions, the sun, moon, and planets spend a considerable fraction of the time at altitudes less than 5°; there are blank spaces available for these data. The second change would be to replace the refraction tables given in the front by others especially prepared for the conditions of temperature and barometric pressure commonly found in the polar regions.

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MATHEMATICAL TABLES—ERRATA

References have been made to Errata in "A new approximation to π"; RMT **360** (Ser), **367** (Ostrogradskiĭ), **368** (Patz), **369** (Cunningham), **370** (Vinogradov), **375** (Fisher & Yates, Croxton & Cowden), **376** (Goncharov), **378** (Hastings & Piedem), **382** (McLachlan), **389** (Stoner), **390** (McLachlan), **392** (N. R. C. Canada), **397** (Aquino); UMT **54** (Br. Standards Inst.), **55** (DeMorgan); QR **28** (Gauss).

99. H. Brandenburg, *Sechsstellige trigonometrische Tafel*, Leipzig, 1932, and Ann Arbor, 1945.

This volume was described in *MTAC*, v. 1, p. 387f and errors are listed on p. 388, and in MTE **28**, v. 1, p. 162. Two further errors have been brought to my notice, both by correspondents in Palestine.

(1) Mr. B. Goussinsky, Superintendent of Surveys, points out that the asterisks in the entries for the tangents of 0° 27′ 40″ and 0° 27′ 50″, p. 32, should be deleted. In the 7-figure tables tan 0° 27′ 30″ is given as 0.007 9996; this has been rounded off (correctly) to 0.008 000 when forming the 6-figure tables, thus changing a leading figure and removing the necessity for asterisks. I have verified that there are no other similar instances of this fault.

(2) Mr. Alexander Katz, of Jerusalem, finds that on p. 23, cot 2° 46′ 32″, for 62694 we must read 62684. This error has taken "evasive action" on several occasions. It occurs in both editions of Brandenburg's 7-figure tables. It is marked in my 1923 edition, and was presumably communicated to Brandenburg but does not appear in an extensive list of corrections that he prepared in 1927. It still occurs in the 1931 edition, and is marked in my copy, but not included in the list that I gave in a review in *The Observatory*, v. 54, 1931, p. 301–302. It was, however, communicated to Brandenburg, and is included in a list of errors published by him in 1932. Nevertheless Brandenburg repeats the error in his 6-figure table in 1932, and my proof readers, who examined the table, failed to find it!

L. J. C.

Editorial Note: For a reference to still other Brandenburg errors, see *MTAC*, v. 2, p. 46f. In the 1923 edition of Brandenburg's 7-figure table are a two-page "Druckfehler-Verzeichnis," and a leaf headed "Berichtigungsbogen zum Überkleben der Druckfehler." In addition to these there is bound with the Brown University copy of these tables purchased in 1929, a 4-page "Nachtrag zum Druckfehler-Verzeichnis," on differently colored paper, dated 15 March 1927, and listing a very large number of errors, and the names of nine individuals assisting in its compilation.

In 1932 the publishers distributed a one-page "Verzeichnis der Druck-, und Formfehler," 18 errors, in the 1931 edition of Brandenburg's 7-figure table. In this sheet, no. 17, *for* 510 lies 519, *read* 310 lies 319.

The five errors in this edition of Brandenburg's 7-figure table, listed by L. J. C. in *The Observatory*, are as follows: p. 95, diff. following sin 4° 59′ 50″, *for* 485, *read* 483; p. 111, cot 7° 39′ 30″, *for* 8870, *read* 8871; p. 157, cot 15° 13′ 30″, *for* 2744, *read* 2743; p. 177, diff. following sin 18° 32′ 10″, *for* 461, *read* 459; p. 335, cot 44° 53′ 0″, *for* 0808, *read* 0807. The first is not included in the 1932 "Verzeichnis."

To conclude the references in *MTAC* to all known errors in the 1931 and 1932 Brandenburg tables we may note that the value of e in each of these tables is given to 30D as, "e = 2.718 281 828 459 045 235 339 784 490 662 . . ." which is correct to only 19D. The value is obtained as the sum to 30D of terms $1/n!$ plus unity, $n = 1(1)20$. Of the values of these 20 terms 4 are not rounded off, and 5 are so rounded. Underneath Brandenburg's incorrect value of e is given Euler's correct value to 23D, taken from his *Introductio in Analysin Infinitorum*, v. 1, § 122. Quite astonishingly the author seems to suggest that he, rather than Euler, is correct.

100. FMR, *Index*. See *MTAC*, v. 2, p. 13–18, 136, 178–181, 219–220.

The notes made by S. A. J. and myself in MTE **89**, p. 178–181, were mainly a budget of suggestions on minor questions for consideration in a possible new edition. Doctors Fletcher and Miller have communicated to us certain statements which should have equal publicity.

P. 35, **3.14** we suggested "27 dec. Thoman" instead of "20 dec. Thoman." Dr. F. writes: "As its title implies, Thoman's book mostly gives 27 dec. But log factorials are to 20 dec.—I have the book in front of me. One or two other tables in it are also to less than 27 dec."

P. 51, **4.41**, *for* 4.412, *read* 4.4121; *for* 4.413, *read* 4.4132. Dr. M. writes "Delete this item. Such an alteration would destroy the effect intended. A_n and S_n are essentially the same function, and the special relations for functions of these types make $2x + 1$ and $x - \frac{1}{2}$ or $2x - 1$ essentially the same, or closely related, arguments. Thus, I regard 4.4122 as being correctly included in the general heading S_n. Likewise 4.4131 and 4.4132, since $x^2 + x = (x + 1)^2 - (x + 1)$, and S_n and V_n are related."

P. 144. I had written, "One wonders at the omission in **8.4** of a reference to Legendre's table of log tan $(45° + \frac{1}{2}x)$." But there is no such table by Legendre; the table is of ln tan $(45° + \frac{1}{2}x)$, which FMR carefully list on p. 184, as Dr. F. has pointed out. Here is a case where an index of names in Part I of the *Index* would have saved me from this slip. While my admiration for the *Index* has increased rather than diminished since I wrote RMT **233**, I have more than once found difficulty in determining what printed or ms. tables by a given author are referred to in the volume. Hence I have started the preparation of a complete card catalogue of names mentioned up to p. 373 of the *Index*. It will only be after its completion that I can feel that all the resources of the volume are at my command. Perhaps these remarks may suggest to the authors some amplification in a second edition.

P. 200, We wrote "7 dec. Brownlee 1923 (Russell, which one?)," since two Russells are mentioned in Part II. Dr. M. comments: "We do not pretend to give all *computers* of tables, listed under other authorship in Part II. The answer to 'which one?' here is, the one mentioned in Brownlee 1923." This remark concerning FMR editorial policy would also make unnecessary the reference to L. Brockway under Sherman 1933, p. 433.

P. 377. I raised the question of a possible edition of Bertrand's *Calcul d. Prob.* in 1888. Dr. F. reports that such an edition is in the library of the University of Liverpool.

P. 420, I noted, "The title of Newton's work is *Trigonometria Britanica* (not *Britannica*)." Dr. F. writes: Spelling 'Britannica' was intentional as (i) it is the correct spelling of the Latin adjective in question, (ii) Newton (as Mr. Cosens pointed out to me) used it in headings, etc. There seems to be one *n* in the title (as you say) because there was no room for two. I dislike departing from the title page, but on this occasion decided, rightly or wrongly, to take a broad view."

P. 431. I had noted that of Schrön 1860, there was an Italian edition in 1867 and a French edition in 1891. Dr. F. remarks that at the University of Liverpool there is a Swedish edition, ed. by F. W. Hultman, Braunschweig, Vieweg, and Stockholm, Bonnier, 1868.

P. 434. I raised the question of listing the 1819 edition of WILLIAM SPENCE, *Mathematical Essays*, either in addition to, or in place of this the listed 1820 edition. Dr. F. writes: "We have seen only Spence 1809. Very interested in your 1819 copy. I had *four* references to 1820: (i) *English Catalogue of Books*, 'c. 1820'; (ii) *Edinburgh Univ. Lib. Cat.;* (iii) *Univ. Coll.; London, Lib. Cat.;* (iv) De Morgan, *The Differential and Integral Calculus*, London, 1842, p. 658–659. (ii)–(iv) give 1820 without 'circa.' " I find that the *Cat. of the Astor Library* in New York also lists a copy of the 1820 edition. Thus four libraries having the 1820 edition at the times their catalogues were published, are listed. On the other hand the Boston Public Library and the Harvard University Library have copies dated 1819, "London, Printed for Thomas and George Underwood." The *Essays* were edited by Sir John F. W. HERSCHEL[1] as a young man, and his "Preface to the Essays" is dated "Slough Dec. 10, 1818"; hence one would expect this volume to have been published in 1819. Since the above was written Dr. F. reported, "We have now seen the Royal Astronomical Society copy of that date."

R. C. A.

[1] HERSCHEL, CHARLES BABBAGE, and GEORGE PEACOCK while still undergraduates formed in 1812 an Analytical Society, which through varied publications did important work in bringing about reforms in mathematical notation; see W. W. R. BALL, *A History of the Study of Mathematics at Cambridge*, Cambridge. 1889. "The Analytical School," p. 117f.

101. NBSMTP, *Table of Circular and Hyperbolic Tangents and Cotangents for Radian Argument*, 1943. See *MTAC*, v. 1, p. 178f.

For the value of tan 1.5708 *read* − 272241.80841, *not* − 27224.18084.

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102. D. H. LEHMER, "On the converse of Fermat's theorem," *Amer. Math. Mo.*, v. 43, 1936, p. 347–354.

This paper contains a table (p. 349–351) of composite solutions n of the congruence $2^n \equiv 2 \pmod{n}$ having prime factors exceeding 313. A recent recomputation of this table by POULET reveals the following complete list of errata:

Delete,	68462551	5851	76839733	1019.
Insert,	44070841	2113	74874869	3533
	70541099	4643	92438581	3331
	71079661	3187	96135601	881.
	74705401	3529		

A relatively unimportant error on p. 351 may be cited. Line 13 purports to give the product of all primes p where $17 \leqslant p \leqslant 101$. This number contains the factor 79^2 and so should be replaced by

$$7754324487462449580421688873809769.$$

D. H. L.

103. J. T. PETERS, *Sechsstellige Tafel der trigonometrischen Funktionen . . . von zehn zu zehn Bogensekunden . . .*, Berlin, 1929; there was a second edition in 1939. See *MTAC*, v. 1, p. 121, 162.

The following additional errors have come to light when preparing copy for the new Chambers' six-figure tables. All are on p. 8 of the 1929 edition.

Cot 0° 27′ 3″ *for* 127.086, *read* .086
4 *for* 126.008, *read* 127.008
5 *for* .930, *read* 126.930.

In other words the integers 127 and 126 each need to be lowered one line.

L. J. C.

104. J. T. PETERS, *Zehnstellige Logarithmentafel. Erster Band: Zehnstellige Logarithmen der Zahlen von 1 bis 100 000*. Berlin, 1922. See *MTAC*, v. 1, p. 57–59.

The proofs of a new six-figure table now being prepared for Messrs. Chambers were compared with this table, revealing, to our great surprise, two errors:

P. 406 log 69731 *for* 843 4358 934, *read* 843 4258 934;
p. 566 log 93748 *for* 974 9620 114, *read* 971 9620 114.

This comparison would not reveal errors in decimals beyond the sixth.

About 1924 I noted that Peters, in spite of the great pains he took to ensure complete accuracy in the tenth decimal (see his Einleitung, p. vii), and his list of cases in which 15 or 16 decimals were necessary for this purpose, had missed one such case,

P. 16 log 11275 *for* 506, *read* 505;

the differences are also affected. This error was pointed out as long ago as 1872 by GLAISHER

(R. A. S., *Mo. Not.*, v. 32, p. 258—misprinted 358), who gives the mantissa as .05211 65505-49998 14···. In the following year Glaisher quoted a letter from the then owner of MICHAEL TAYLOR's copy of VLACQ (*ibid.*, v. 33, p. 452) saying that some previous owner (GARDINER is suggested) had corrected log 11275 by hand.

Glaisher's remarks on end-figure errors are quoted in N **72**. They were prompted by this particular "error."

L. J. C.

UNPUBLISHED MATHEMATICAL TABLES

53[A, B].—NBSMTP, *Tables of Circumferences and Areas of Circles.* Tables prepared for the U. S. Bureau of Ordnance, Navy Department. Compare, *MTAC*, v. 2, p. 86–87.

These tables are for circles with diameters ranging [.001(.001)10; 6D]. The computations were made with IBM equipment, and a manuscript was prepared on the tabulator.

NBSMTP

54[L, M].—CARL HAMMER, *Table of selected values of* $Li(x) = \int_0^x dt/\ln t$, and $\int_2^x dt/\ln t$, mss. in possession of the author at 304 West 105th St., New York City; and in the Library at Brown University.

These tables are for x = [2(1)10(10)100(100)1000(1000)10 000(10 000)100 000; 8S]. The values for $Li(x)$ were previously given by J. VON SOLDNER, *Théorie et Tables d'une Nouvelle Fonction Transcendante*, Munich, 1809, p. 43–49, for x = [0(.01)1(.1)2(.5)3(1)20; 7D], [22(2)40(5)80(10)160(20)320(40)640(80)1280; 8S], Δ^3 to .8. This table was reprinted (without Δ^3, and with a misprint of 1220 for 1280) in A. DEMORGAN, *The Differential and Integral Calculus* . . ., London, 1842, p. 662–663. Thus of 90 values given in the ms., 22 were Soldner's values. Three other values of $Li(x)$, for $x = 10^3, 10^4, 10^5$ were taken from F. W. BESSEL, "Untersuchung der durch das Integral $\int dx/\ln x$ ausgedrückten transcendenten Function," *Königsberger Archiv f. Naturw. u. Math.*, v. 1, 1811, p. 31, and F. W. BESSEL, *Abhandlungen*, Leipzig, v. 2, 1876, p. 339. The other 65 values were computed by means of NBSMTP, (a) *Table of Natural Logarithms*, v. 2, 1941; (b) *Table of Sine and Cosine Integrals* . . ., 1942; (c) *Table of Sine, Cosine and Exponential Integrals*, 2 v., 1940; (d) *Tables of Lagrangian Interpolation Coefficients*, 1944, using five points for x = 500 to 9000, and seven points for x = 20 000 to 90 000.

C. HAMMER

55[P].—SIDNEY JOHNSTON, *Roller Chain Transmission Basic Exact Centre Distance Tables.* Ms., iii + 10 sheets typed on one side. 20.3 × 32 cm. Original in possession of the author at 81 Fountain St., Manchester 2, England; carbon copy in the Library of Brown University. Among the "References" in the ms. are the following: (a) K. B. JACOB, "Driving chains and theory application to power transmission," Engineering and Shipbuilding Draughtsmen's Assoc., *Trans.*, 1928–29 (also as a pamphlet, T. 5, centre distance tables, p. 47–54); (b) *Machinery's Handbook*, New York, Industrial Press, twelfth ed., 1943, p. 861–2.

These tables are intended to serve the mechanical engineer in solving the bothersome problem of the design of roller chain transmission. Suppose that a roller chain of N links

and pitch P connects two chain gears, the number of whose teeth are T and t. The tables enable one to find the distance S between the centers of the gears. The formula on which the tables are based is:

$$S = \tfrac{1}{2}P(D - d)\csc x$$

where

$$D = \csc \pi T^{-1} \quad \text{and} \quad d = \csc \pi t^{-1},$$

and x is the least positive solution of

$$\cot x + cx = a, \quad \text{where}$$
$$c = \pi^{-1}(T - t)/(D - d) \quad \text{and} \quad a = [N - \tfrac{1}{2}(T + t)]/(D - d)$$

The table is one of double entry giving the factor $\csc x$ to 6, 5 and 4D in terms of c and a. The value of c in a typical case is slightly greater than unity. The present tables are for $c = 1$, 1.01 and 1.02 only. The second variable a has the following range

a = [1.75(.002)1.8(.0025)1.835(.005)1.93(.01)2.07, 2.075(.0125)2.325(.025)2.95(.05) 4.15, 4.125(.125)6.5(.25)10; 6D], [11.25(1.25)22.5(2.5)35(5)105; 5D], [100(50)300(100)1000; 4D].

The partial differences of $\csc x$ with respect to c and a are given together with the Bessel coefficient $\Delta(\Delta - 1)/4$. There is an auxiliary table of $\csc \pi T^{-1}$ for T = [10(1)160; 8D] for finding the pitch diameters d and D. Comparing this table with a similar smaller table to 4D in British Standards Institution, *Specifications for Steel Roller Chains and Chain Wheels*, revised April 1934, no. 228–1934, p. 18–19, one finds in the latter table, 10 last-figure unit errors: in excess for T = 44, 46, 63, 76, and in defect for T = 11, 22, 91, 95, 107, 133; also at T = 127, *for* 49.4295, *read* 40.4295.

The present tables, if made available to machine design people, should do much to replace the crude approximations usually resorted to in dealing with this comparatively precise problem. A few handbooks give the exact formula

$$N = T + \pi^{-1}(T - t)(\tan A - A)$$

where

$$A = \arccos P(T - t)/2\pi S$$

or an equivalent formula but the reviewer has not found any other tables for obtaining S directly.

D. H. L.

MECHANICAL AIDS TO COMPUTATION

The reader is referred to the first two articles of this issue, dealing with MAC topics.

28[Z].—Akad. N., SSSR, Moscow, *Izvestiia͡*, *Otdelenie Tekhnicheskikh Nauk*, 1946, no. 8, September, p. 1065–1200. 16.7 × 25.8 cm. Entirely in Russian.

This issue is almost wholly devoted to material dealing with computation and computing mechanisms. The contents are as follows:

N. G. Bruevich, "The present state and problems of the theory of mechanism precision," p. 1065–1079.

I. I͡A. Akushskiĭ, "An outline of punched cards machines," p. 1081–1120.

L. I. Gutenmakher, "Electrical models of physical phenomena and their applications in technology and physics," p. 1121–1146. In the literature list there are several references to American authors including two to Bush.

L. Li͡usternik, "Problems of computational mathematics," p. 1147–1156.

L. ÎA. NEĬSHULER, "Tabulation of functions," p. 1157–1176. There are here two references (p. 1157, 1175) to *MTAC* and to U. S. A. as "the country with the greatest development in the industry of calculating machines."

CHRONICLE: M. L. BYKHOVSKIĬ, "The new differential analyzer of Bush," p. 1177–1198. An illustrated description based on the long article of V. BUSH & S. H. CALDWELL, "A new type of differential analyzer," Franklin Inst., *Jn.*, v. 240, 1945, p. 255–326; see *MTAC*, v. 2, p. 89–91.

R. C. A.

NOTES

68. DOCTOR COMRIE'S ADDRESS.—We regret that we omitted to state in connection with the L. J. C. article, published v. 2, p. 149–159, which has been much in demand, that it was the address which he delivered 31 October 1945 at the Conference on Advanced Computation Techniques (*MTAC*, v. 2, p. 65–68), as chairman of subcommittee *Z* of the Committee on Mathematical Tables and Other Aids to Computation.

69. GIBBS' PHENOMENON.—I feel that the Note on the sine integral in *MTAC*, v. 2, p. 195, will give the impression that any description of the Gibbs' phenomenon in which the number 1.08949 (approx.) occurs is wrong, and that this number should be replaced by 1.17898 (approx.) as the result of a new and careful evaluation of $K = (2/\pi)\mathrm{Si}\,\pi$. This is not so, as the appropriate number depends upon the way the phenomenon is described.

The series $F(t)$ represented by $\frac{1}{2}(\pi - t)$ for $o < t < \pi$, and by $-\frac{1}{2}(\pi + t)$ for $-\pi < t < o$ [not by $\frac{1}{2}(t - \pi)$ as stated in the Note if we interpret t algebraically] is an odd function with a discontinuity or jump of π from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$ at $t = 0$. The Fourier series representing this function, i.e.,

$$\sum_{n=1}^{\infty} \sin nt/n,$$

exhibits the Gibbs phenomenon as an overshoot at each end of amount, say, δ, and, measured from the origin, the function jumps each way by an amount $\frac{1}{2}\pi + \delta$ which is given by

$$\mathrm{Si}\,\pi = \int_0^{\pi} \sin t\, dt/t = \tfrac{1}{2}\pi + \mathrm{si}\,\pi,$$

where $$\mathrm{si}\,\pi = -\int_{\pi}^{\infty} \sin t\, dt/t \sim .28114, \quad \text{so that} \quad \delta = \mathrm{si}\,\pi.$$

The phenomenon may be defined as in *MTAC* by the ratio K of $\frac{1}{2}\pi + \delta$ to $\frac{1}{2}\pi$, i.e., $K = (2/\pi)[\frac{1}{2}\pi + \mathrm{si}\,\pi] = 1 + (2/\pi)\,\mathrm{si}\,\pi \sim 1.17898$. This ratio is also that of the jump including both the overshoots to the jump itself, i.e., $\pi + 2\delta$ to π.

We can, however, define the phenomenon by the ratio K' of the jump + either overshoot to the jump itself, i.e., $\pi + \delta$ to π, so that

$$K' = (1/\pi)(\pi + \delta) = 1 + (1/\pi)\,\mathrm{si}\,\pi \sim 1.08949.$$

When the jump is not necessarily at the origin nor of amount π symmetrically disposed about the t axis, it is usual to describe the phenomenon as an overshoot at each end by an amount which is about 9% of the jump

itself,[1] a definition which follows naturally from the ratio K' above. (See, for instance, T. v. KÁRMÁN & M. A. BIOT, *Mathematical Methods in Engineering*, New York and London, 1940, p. 335). The ratio K is equivalent to saying that the sum of the two overshoots is about 18% of the jump.

K. KNOPP in his *Theory and Application of Infinite Series* (London, 1928) stated on p. 380 [p. 392 of the 1931 German ed.] that the first maximum has a value $\frac{1}{2}\pi(1.08949)$ i.e. our K', whereas his diagram on p. 379 clearly suggests that the value is really K. Obviously somewhere in the literature there has been a confusion between the definition of the overshoots as a percentage of the jump, and Knopp has used the correct diagram with a wrong description.

It is useful therefore to draw attention to the matter, but the Note in *MTAC* has obscured the issue by relating it to the accurate computation of the sine integral. Actually, to explain the difference between K and K' there was no need to use the Harvard Automatic Sequence Controlled Calculator and prove $K = 1.17897975. \ . \ . \ .$ If we eliminate si π between K and K', we have $K' = \frac{1}{2}(K + 1)$, so that we can obtain the interesting relation referred to in the footnote to the article in *MTAC*, without having to know the value of si π at all.

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[1] This result was stated by M. Bôcher, *Annals Math.*, s. 2, v. 7, 1906, p. 131.—EDITOR.

NOTE BY R. C. A.: We are glad to have Mr. Millington's communication which will doubtless interest many readers. It seems desirable, however, to make clear that justification for his sweeping first paragraph (to which he returns in the last) is doubtful. I now refer to the following five places (which are the only ones) of my Note where the 1.08949 is involved: (i) Zygmund states that $K = 1.089490$, which is, of course, an error; (ii–iii) the two Szász papers before 1944 (Amer. Math. Soc., *Trans.*, v. 53, 1943, p. 440, and v. 54, 1943, p. 497) where Zygmund's incorrect value is copied. (iv) HARDY & ROGOSINSKI state that $k = \int_0^\pi \sin t\, dt/t = 1.71 \ . \ . \ .$ (when it should have been $1.85 \ . \ . \ .$) which is really equivalent to Zygmund's erroneous statement, as I pointed out. There remains, then, one and only one statement involving 1.08949, namely: (v) that of ZALCWASSER, who obtains the limit $\frac{1}{2}\pi(1.089)$ which, as we remarked in the Note, is exactly the error of HARDY & ROGOSINSKI. Thus in every one of the "five places", indubitable errors were listed. In not one of these errors enters the question of misinterpretation of "the way the phenomenon is described".

70. THE GRAEFFE PROCESS.—In view of our previous articles on this subject by D. H. L. "The Graeffe process as applied to power series," *MTAC*, v. 1, p. 377f, and by Mr. MITCHELL, "The Graeffe process," *MTAC*, v. 2, p. 57f, the following references to three articles published elsewhere may be noted: E. BODEWIG, "On Graeffe's method for solving algebraic equations," *Quart. Appl. Math.*, v. 4, 1946, p. 177–190; JOSE L. MASSERA, "El método de Gräffe para resolver ecuaciones algebraicas," Montevideo, Universidad, Facultad de Ingenieria, *Boletin*, año 10, v. 3, Dec. 1945, p. 1–20; R. SAN JUAN, "Complementos al método de Gräffe para la resolución de ecuaciones algébricas," *Revista Matem. Hispano-Americana*, s. 3, v. 1, 1939, p. 1–14.

71. WAS THERE AN ITALIAN REPRINT OF VEGA'S *Thesaurus* AFTER 1896?.—We may begin by quoting the following five authorities which state that there was: (**a**) In *Jahrb. ü.d. Fortschritte d. Math., 1910*, p. 1054, is the entry,

"G. Vega, Thesaurus, logarithmorum completus. Vollständige Sammlung grösser logarithmisch-trigonometrischer Tafeln. Neudruck. Mailand. 684 S. 4°." (**b**) H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales*, Paris, 1911, p. vi; in listing the photozincographic reproductions of the *Thesaurus*, Istituto Geografico Militare of Florence, is the statement "un troisième tirage vient d'être effectué (1910)." (**c**) *Modern Instruments and Methods of Calculation*, ed. by E. M. HORSBURGH, London, Bell, and Royal Soc. of Edinburgh, [1914], p. 50, 52, "reprinted Milan, 1909." (**d**) F. J. DUARTE, *Nouvelles Tables Logarithmiques* . . ., Paris, 1933, p. xxiv, apparently quotes (b) as his authority for a third reprint by the Istituto in 1910. (**e**) FMR, *Index*, 1946, p. 440, lists, as in (b) and (d) a third Istituto reprint of 1910. A possible explanation of the 1909 date in (c) is that there was confusion with the sixteenth edition of Cremona's Italian translation in that year, of Vega's *Manuel logarithmique et trigonométrique* (see *Intern. Cat. Sci. Lit.*, v. 9A, p. 125.

In spite of such an array of authorities I was puzzled that I could find (i) no mathematical bibliography except (a), and no Italian bibliography, which listed an Italian edition after 1896; (ii) not a single library which had a copy, and (iii) no review in any periodical. Hence I wrote to the Director of the Istituto Geografico Militare offering to purchase a copy of the 1909 or 1910 edition. In his reply dated 2 Oct. 1946 occurs the following paragraph, in translation: "As a matter of fact besides not possessing any copy of the 1910 edition of Vega's Thesaurus we cannot assure you that such a copy was an exact reprint of the 1896 edition, since the documents relative to it in the Library were destroyed by the Germans. From oral testimony gathered from clerks who were more or less directly concerned with the reprinting of the Thesaurus it would seem that the 1910 edition would have been in complete conformity with the 1896 edition." As a result of this somewhat inconclusive paragraph, I published the rather vague statement about edition **4**, *MTAC*, v. 2, p. 163.

Shortly after reading this L. J. C. sent me a copy of a translation of a statement which he had received from the Istituto during 1922–24 (when he was on the staff of Swarthmore College), in reply to his varied queries including one about a 1909 or 1910 edition. The following sentences there occur: "No other reprint of these tables has been issued by this Institute since the issue of the 1896 edition, and there is no record of any edition published in Milan in 1908 or subsequent to that date." "The zincographic plates prepared for the reprint are still preserved in the Institute." L. J. C. also drew my attention to his published statement in the *Observatory*, v. 52, 1929, p. 325 about these plates not being used for any edition subsequent to 1896. Hence I am now inclined to subscribe to the following statement by L. J. C. in a personal letter: "I will not believe in the 1909 or 1910 edition until I have an affidavit from somebody who has seen one."

R. C. A.

72. WHAT IS AN ERROR?—Those who delight in pointing out trivial end-figure errors may like to be reminded of the following words of wisdom from Glaisher's pen (R. A. S., *Mo. Not.*, v. 32, 1872, p. 261): "The increase of the last figure in tables, when the succeeding figures are greater than 500. . .,

seems to deserve more attention than it has received. Errata, such as some noticed in this communication, where the succeeding figures are 499. . ., are by no means uncommon; and it appears that the discoverers of them imagine they are doing some service by noting them. Take, for example, one of the cases in this note: the figures starting from the tenth are 5 49998. . .; if we take 5 as the tenth figure, the error is 49998. . ., if 6, the error is 50002, differing by 00004. Now, as our table only professes to give 10 places correctly (regard being paid to the magnitude of the figure in the eleventh place) a difference in the fifteenth place does not come in question at all: 5 and 6 are both equally correct; they only differ by quantities, which throughout all the rest of the table we agree to neglect. It is a matter of regret that all such valueless refinements are not avoided by the author always explaining the exact convention on which the last figure is increased. A very convenient arrangement would be to understand that when x figures of a number were tabulated, the error was less than 6 in the next figure; or, if the calculator wished to be more accurate, 56 in the next two figures. To obtain a table of x figures, it is usual to calculate $x + 1$, or $x + 2$ figures, and the inconvenience of extending the calculation further in the particular case when the next figure is 5, or the next two 50, is, in many cases, excessive, and as the result is of no additional value when obtained, a figure "wrong" under these circumstances ought not to be styled an error. Probably a good many of Lefort's errata are of this class. Babbage, in the introduction to his well-known table of seven-figure logarithms, states, that in ninety-three instances the next three figures in Vega were 500, and that in all these cases the logarithms were carried to more than ten places to determine whether the figures were really 500. . . or 499. . ., and decide whether the least figure was to be increased or not. This appears to me to have been quite needless. It sets up an unnecessary and artificial standard of accuracy for the numbers whose seventh, eighth, and ninth figures happen to be 4, 9, 9 or 5, 0, 0. To the user of a table of seven-figure logarithms it is a matter of really no importance whether his error is 499 or 501; he is content to make an error of 5, and an additional error of $\pm$ 0.01 is of no consequence."

With this I thoroughly agree—so much so that on more than one occasion I have written to our beloved editor saying "I have found . . . errors of less than one unit in . . . tables, but am not sending them to you, lest you should be tempted to publish them." In the Introduction to the BAASMTC, *Mathematical Tables*, v. 6, *Bessel Functions, Part I*, which I edited, I wrote: "In general all values have been computed to two decimals more than are given in these tables; the error of any tabulated value should not exceed $\pm$ 0.52 units of the last decimal." Incidentally Professor H. H. AIKEN informs me that he has not found any error exceeding my assigned limits in the functions J_0 and J_1. It would be futile to go to the trouble of altering any end figures where the error lies between $\pm$ 0.50 and $\pm$ 0.52, since an interpolate is liable, in any case, to be in error by at least a whole unit.

Errors, including those in the last decimal, often enable the table detective to ascertain how a table has been computed. One has only to instance the case of Buckingham, who refused to give any information about the compilation of his eight-place tables. But he was hopelessly betrayed by his errors (see *MTAC*, v. 1, 1943, p. 88f). The 2000 errors (exclusive of those

of a unit in the last decimal) in HAYASHI, ***Siebenstellige . . . Tafeln*** (1926) give ample proof (a) that he did not check by differencing and (b) that he used a building-up process for intermediate values. Curiously enough, his end figures are fairly reliable. A run of errors in an early volume of DAVIS[1] showed (a) that he had made independent subtabulations in each interval, (b) that he had relied on repetition—the poorest possible check, and (c) that he had not checked by differencing. Other errors showed (d) that he had neglected second differences when interpolating 10-figure logarithms, and (e) that he had taken 10-figure logarithms of rounded-off quantities containing only five or six significant figures. But to his credit be it said that he was an apt "pupil" and can be trusted not to fall into any of these traps again!

Mrs. GIFFORD's end figures,[2] especially in the tangents, show the neglect of higher order differences; at one point there is a perfect wave in each 10″ interval, with an amplitude of 3 units. The observation that her sines near 90° were often in error by 99, 100 or 101 units led to a confession (in the true Sherlock Holmes style) that she "pre-fabricated" the first six decimals, and later added the seventh and eighth, with the not unnatural result that the sixth is often one out!

DUFFIELD's claim to have computed his logarithms to 12 decimals, increasing the tenth when the last two were 50 or more, is immediately shown to be false by the fact that his end-figure errors are (with a few exceptions, which can be accounted for) the same as those of Vega![3]

The fact that BENSON had copied from BRANDENBURG was revealed by his end-figure errors. He, too, was forced into a confession (*MTAC*, v. 1, 1943, p. 9) that shows he had not been honest either in his compilation, or in his preface. IVES, who also wrote a deceitful preface, provided at least a part of the clue to his plagiarisms by his errors.[4]

I have seen a 5-figure navigational table which contained just five per cent of end-figure errors, because it had been prepared from a six-figure table, but rounding off all 5's in the *same* direction.

L. J. C.

EDITORIAL NOTES: In *MTAC*, v. 1, p. 144 and 58, accuracy of half a unit in the last decimal place by PETERS, and PETERS & STEIN has been noted; and also on p. 145 accuracy less than .502 in the last decimal place. See further "Cayley and tabulation," p. 98. In the quotation of a passage from Glaisher's pen "Lefort's errata" are those referred to in *MTAC*, v. 2, p. 164–165. The well-known 7D table of CHARLES BABBAGE is ***Table of the Logarithms of the Natural Numbers from 1 to 108 000***, London 1827, and various later editions.

[1] H. T. DAVIS, ***Tables of the Higher Mathematical Functions***, v. 1. Bloomington, Ind., 1933.

[2] E. GIFFORD, see *MTAC*, v. 1, p. 11, 24f, 64f.

[3] See *MTAC*, v. 2, p. 164.

[4] H. C. IVES, see *MTAC*, v. 1, p. 9f.

QUERIES

21. PORTRAITS AND BIOGRAPHIES OF BRITISH MATHEMATICAL TABLE MAKERS.—Where may portraits be seen, or copies possibly be procured, of any of the following individuals: PETER BARLOW (1776–1862), HENRY BRIGGS (1561–1630), OLIVER BYRNE (publs. of 1838–77), ALLAN JOSEPH CHAMPNEYS CUNNINGHAM (1842–1928), JAMES DODSON (d. 1757), RICHARD

FARLEY (publs. of 1840–56), HERSCHELL E. FILIPOWSKI (publs. of 1849–57), EMMA GIFFORD (1861–1936), PETER GRAY (1807?–1887), EDWARD LINDSAY INCE (1891–1941), HENRY SHERWIN (publs. of 1705–1741), ROBERT SHORTREDE (1800–1868), JOHN SPEIDELL (publ. of 1619), MICHAEL TAYLOR (1756–1789)? None of them are listed in Ball's Collection of portraits (E. M. HORSBURGH, *Modern Instruments and Methods of Calculation*, London, 1914). Karl Pearson tells us (*Logarithmetica Britannica*, part IX, 1924) that he vainly sought a portrait of Briggs. The *Dict. Nat. Biog.* contains sketches of Briggs and Gray, and there is biographical material about Cunningham in (a) Poggendorff, (b) London Math. Soc., *Jn.*, v. 3, 1928 (by A. E. Western), and about Ince in (a) *Who's Who 1941*, (b) London Mathem. Soc., *Jn.*, v. 16, 1941 (by E. T. Whittaker), (c) Edinb. Math. Soc., *Proc.*, s. 2, v. 6, 1941 (by A. W. Young), (d) *Nature*, v. 148, 1941 (by A. C. Aiken). Where may one find biographical data concerning the other persons listed?

R. C. A.

QUERIES—REPLIES

28. TABLES OF TAN$^{-1}(m/n)$ (Q**14**, v. 1, p. 431; QR**18**, v. 1, p. 460; **20**, v. 2, p. 62; **24**, p. 147).—In *MTAC*, v. 2, p. 63, footnote 2, J. C. P. MILLER suggested the following problem: Determine all positive integers N which are fundamental in the sense that there is no relation of the form

$$\tan^{-1} N = \sum_i \lambda_i \tan^{-1} n_i \tag{1}$$

(where the λ_i are integers and the n_i are positive integers *less than* N).

This problem can be completely solved using methods of elementary number theory (C. F. GAUSS, *Werke*, v. 2, 1863, and 1876, p. 477 and 523, and the papers by C. STØRMER and others quoted in *MTAC*, v. 2, p. 28). It can be shown that N is fundamental if and only if $N^2 + 1$ has a prime factor which is not a factor of a number $n^2 + 1$ with $n < N$. An effective construction for the relation of the form (1) in the case when N is not fundamental can be given; a table giving all such relations with N satisfying $N^2 + 1 < 100\,000$ has been prepared. The positive integers which are fundamental form a minimal "integral" basis for the set $\{\tan^{-1} n\}$ and therefore also for the set[1] $\{\tan^{-1}(m/n)\}$. The connection between this "integral" basis and the "rational" basis which was apparently known to GAUSS[2] can be made clear by the following remark. Corresponding to a prime $p = 2$ or $4n + 1$, Gauss has $\tan^{-1}(a/b)$ where a and b are determined uniquely by the conditions $a \geqslant b > 0$ and $a^2 + b^2 = p$ while we have $\tan^{-1} N$ where N is the least positive integer such that $N^2 + 1$ is a multiple of p. A table has been prepared, covering all such primes $p < 500$, expressing $\tan^{-1}(a/b)$ in terms of $\tan^{-1} N$ and $\tan^{-1} m$ with $m < N$. These results will be submitted to the London Math. So., *Jn.*

The reduction of $\tan^{-1}$ 1 40333 78718 was attempted in order to test the reduction algorithm. This is reducible since the prime factors of $1\ 40333\ 78718^2 + 1$ all occur as factors of $n^2 + 1$ with $n < 1\ 40333\ 78718$. Use of the factorisation given by Gauss (*Werke*, v. 2, p. 481) led to a contradiction which was found to be due to an extra factor 13 in the decomposition

given by Gauss. The entry in his table corresponding to 1 40333 78718 should read

$$5\cdot5\cdot13\cdot17\cdot17\cdot61\cdot61\cdot61\cdot61\cdot73\cdot73\cdot157\cdot181$$

and the required reduction is $\tan^{-1} 1\,40333\,78718 = -\tan^{-1} 28 - 2\tan^{-1} 27 + \tan^{-1} 19 - 4\tan^{-1} 11 - \tan^{-1} 5 - 2\tan^{-1} 4 - \tan^{-1} 2 + 20\tan^{-1} 1$.

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[1] D. H. LEHMER, *Duke Math. Jn.*, v. 4, 1938, p. 323–340.

[2] C. F. GAUSS, *l.c.*, p. 523. See C. STØRMER, *Archiv f. Math. og Naturv.*, v. 19, 1896, no. 3, p. 1–96, especially p. 77.

CORRIGENDA

V. 1, p. 360, l. − 4, *for* $x = 1(2)6(1)10$, *read* $x = 1(1)6(2)10$.

V. 2, p. 68, **275**, *for* PEDERSON, *read* PEDERSEN.

V. 2, p. 195, l. 6, *for* $\frac{1}{2}(t - \pi)$, *read* $-\frac{1}{2}(t + \pi)$; p. 228, last line, *delete* "266, *read* 288," *and replace by* "Lewin, *read* Levin."

CLASSIFICATION OF TABLES, AND SUBCOMMITTEES

A. Arithmetical Tables. Mathematical Constants
B. Powers
C. Logarithms
D. Circular Functions
E. Hyperbolic and Exponential Functions
Professor DAVIS, *chairman*, Professor ELDER, Professor KETCHUM, Doctor LOWAN

F. Theory of Numbers
Professor LEHMER

G. Higher Algebra
Professor LEHMER

H. Numerical Solution of Equations
J. Summation of Series

I. Finite Differences. Interpolation
K. Statistics
Professor WILKS, *chairman*, Professor COCHRAN, Professor EISENHART, Professor FELLER, Professor HOEL

L. Higher Mathematical Functions
M. Integrals

N. Interest and Investment
O. Actuarial Science
Mister ELSTON, *chairman*, Mister THOMPSON, Mister WILLIAMSON

P. Engineering

Q. Astronomy
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R. Geodesy
S. Physics, Geophysics, Crystallography
T. Chemistry
U. Navigation

V. Aerodynamics, Hydrodynamics, Ballistics

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EDITORIAL AND OTHER NOTICES

The addresses of all contributors to each issue of *MTAC* are given in that issue, those of the Committee being on cover 2. The use of initials only indicates a member of the Executive Committee.

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CONTENTS

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